V. VDQML: some variations

V.1. Non-rigid terms

Remark. There's an important disanalogy between constants and predicates in VDQML:

(NI) $\vDash_{\text{VDQML}}\ a = b \to \Box a = b$

(NC) $\nvDash_{\text{VDQML}}\ \forall x(Fx \leftrightarrow Gx) \to \Box\forall x(Fx \leftrightarrow Gx)$

- The desirability of (NI) depends on (i) what constants are taken to formalize; (ii) our views on rigidity. Consider:
  (1) Plato = the teacher of Aristotle but he might not have been
  (2) Sue = Arkela but she might not have been
- The source of the difference isn't hard to pinpoint:
  - The extension of a, $\mathscr{I}(a)$, remains fixed from world-to-world
  - The extension of F, $\mathscr{I}_w(F)$, varies from world to world

  Individual constants are 'strongly rigid' in Kripke's terminology

To allow for non-rigid terms, we may modify our model theory as follows:

Definition V.1.1 (Semantics for non-rigid terms)**.** A NRT-VDQML model is quintuple, $\langle \mathscr{W}, \mathscr{R}, \mathscr{D}, \mathscr{Q}, \mathscr{I} \rangle$, just like a VDQML model, except $\mathscr{I}$ assigns each constant α a referent in each world:

- $\mathscr{I}_w(\alpha) \in \mathscr{D}$

We then relativize the denotation of terms, and the satisfaction conditions of atomic formulas to worlds in the natural way:

$$[\alpha]_{\mathscr{M},g,w} = \begin{cases} \mathscr{I}_w(\alpha) \text{ if } \alpha \text{ is a constant} \\ g(\alpha) \text{ if } \alpha \text{ is a variable} \end{cases}$$

- $\mathrm{V}_{\mathscr{M},g}(\alpha = \beta, w) = 1$ iff $[\alpha]_{\mathscr{M},g,w} = [\beta]_{\mathscr{M},g,w}$
- $\mathrm{V}_{\mathscr{M},g}(\Pi\alpha_1, \ldots, \alpha_n, w) = 1$ iff $\langle [\alpha_1]_{\mathscr{M},g,w}, \ldots [\alpha_n]_{\mathscr{M},g,w} \rangle \in \mathscr{I}_w(\Pi)$

The other semantic clauses and the definition of validity remain the same as before.

(NI) $\nvDash_{\text{NRT-VDQML}}\ a = b \to \Box a = b$

(NC) $\nvDash_{\text{NRT-VDQML}}\ \forall x(Fx \leftrightarrow Gx) \to \Box\forall x(Fx \leftrightarrow Gx)$

V.2. Possibilist quantifiers (compare LfP 9.6.4)

Recall that the Barcan formula is not VDQML-valid:

(BF) $\nvDash_{\text{VDQML}} \Diamond\exists\alpha\phi \rightarrow \exists\alpha\Diamond\phi$

(CBF) $\nvDash_{\text{VDQML}} \exists\alpha\Diamond\phi \rightarrow \Diamond\exists\alpha\phi$

This non-validity flows in part from a so-called 'actualist' treatment of quantifiers in VDQML. We could instead introduce a 'possibilist' quantifier $\forall_p$:

'Actualist'	*'Possibilist'*
$\forall$ ranges over $\mathscr{D}_w$ in w	$\forall_p$ ranges over $\mathscr{D}$ in w

The only difference compared with VDQML is the semantic clause for $\forall_p$:

- $\mathrm{V}_{\mathscr{M},g}(\forall\alpha\phi, w) = 1$ iff, for every $\underline{d \in \mathscr{D}_w}$, $\mathrm{V}_{\mathscr{M},g^{\alpha}_{d}}(\phi, w) = 1$
- $\mathrm{V}_{\mathscr{M},g}(\forall_p\alpha\phi, w) = 1$ iff, for every $\underline{d \in \mathscr{D}}$, $\mathrm{V}_{\mathscr{M},g^{\alpha}_{d}}(\phi, w) = 1$

Possibilist quantifiers restore the VDQML-validity of (BF) and (CBF):

(BF) $\vDash_{\text{VDQML}} \Diamond\exists\alpha\phi \rightarrow \exists\alpha\Diamond\phi$

(CBF) $\vDash_{\text{VDQML}} \exists\alpha\Diamond\phi \rightarrow \Diamond\exists\alpha\phi$

V.3. Strict actualist predicates

Another contentious non-validity in VDQML is sometimes called the 'being constraint':

(BC) $\nvDash_{\text{VDQML}} Fx \rightarrow \exists y\, y = x$

One way to render (BC) valid is to switch to a 'strict actualist' semantics for predication. SA-VDQML-semantics makes the following modification to the definition of VDQML model $\langle \mathscr{W}, \mathscr{R}, \mathscr{D}, \mathscr{Q}, \mathscr{I} \rangle$:

- $\mathscr{I}(\Pi^n)$ is a set of $n+1$-tuples of the form $\langle u_1, \ldots, u_n, w \rangle$, where $u_1, \ldots u_n$ are <u>members of $\mathscr{D}_w$</u> and $w \in \mathscr{W}$, for each n-place predicate Π^n

(BC) $\vDash_{\text{SA-VDQML}} Fx \rightarrow \exists y\, y = x$