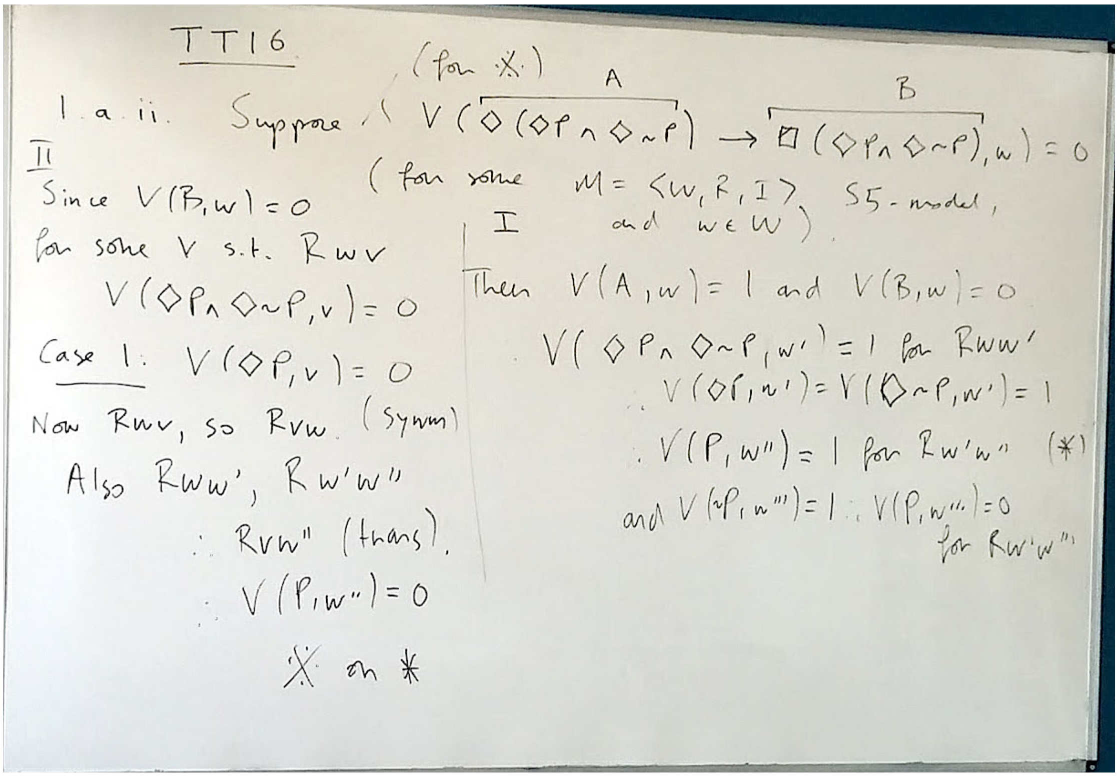
127: TT16 past paper

Board pictures

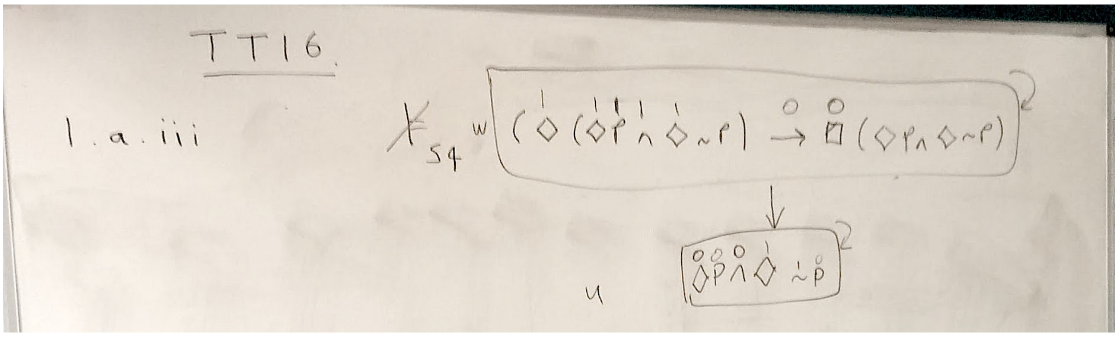
Revision class TT17

Question 1

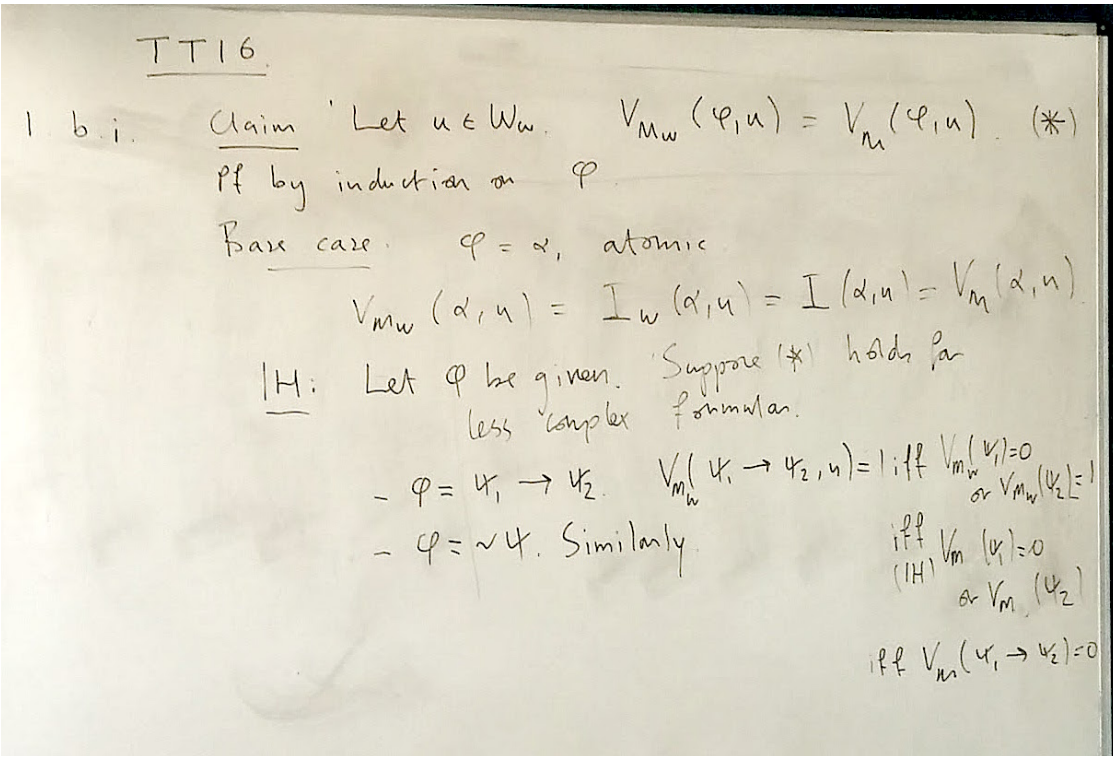
(1.a.ii)



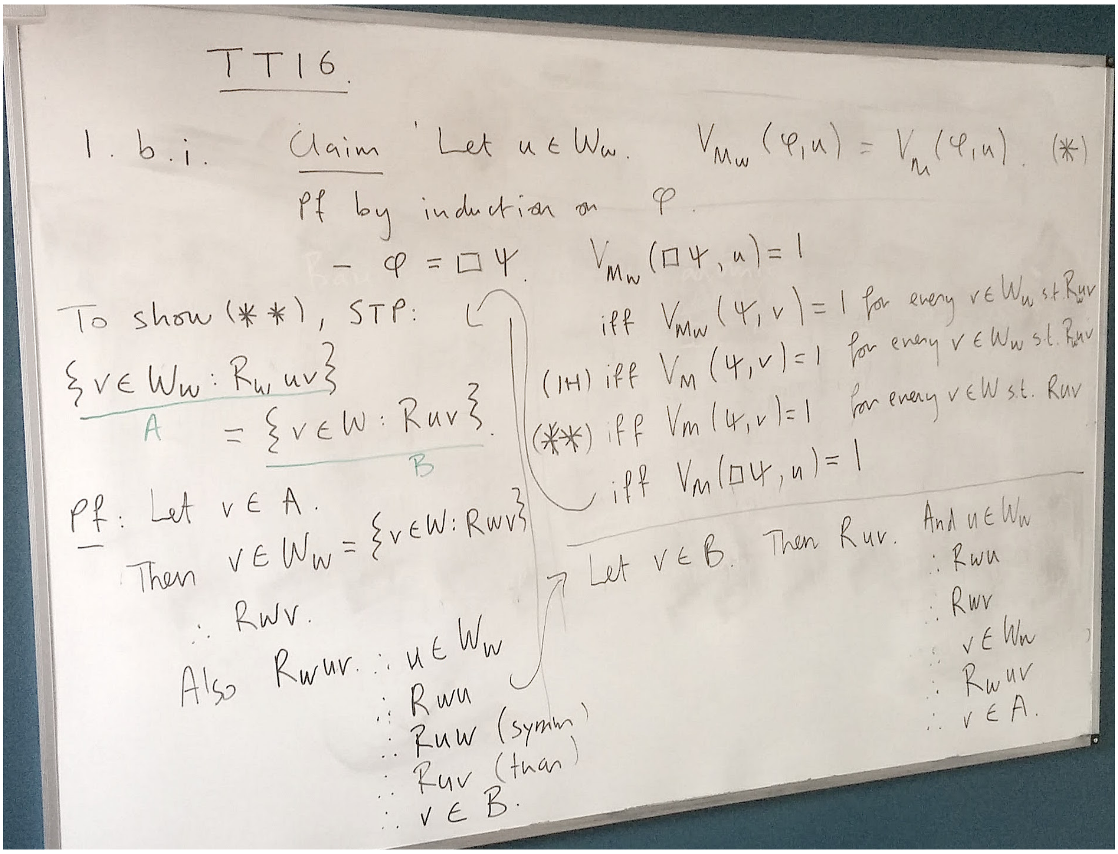
(1.a.iii)


TT16
1.a.iii
$\nvDash_{S4}$ w $(\Diamond(\Diamond p \wedge \Diamond \sim p) \rightarrow \Box(\Diamond p \wedge \Diamond \sim p))$
u
$\Diamond p \wedge \Diamond \sim p$

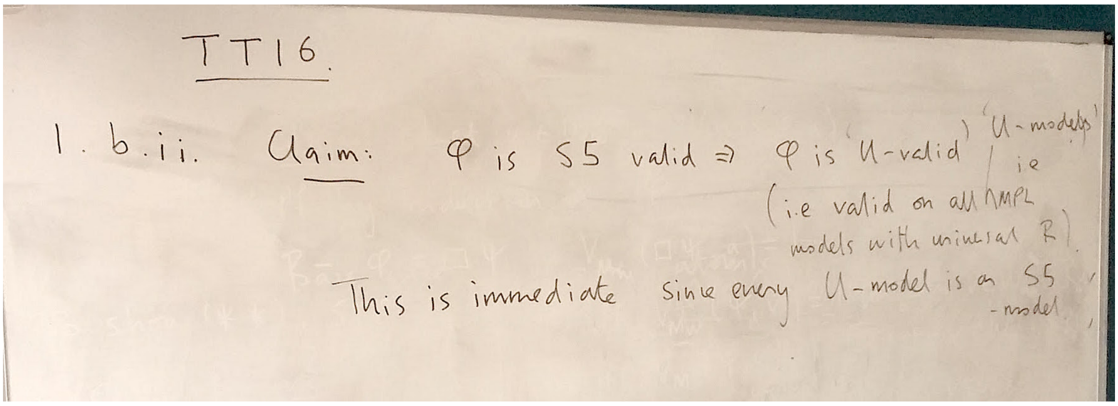
(1.b.i)


TT16
1.b.i. Claim: Let $u \in W_w$. $V_{M_w}(\varphi, u) = V_M(\varphi, u)$. (*)
Pf by induction on φ
Base case: $\varphi = \alpha$, atomic
$V_{M_w}(\alpha, u) = I_w(\alpha, u) = I(\alpha, u) = V_M(\alpha, u)$
IH: Let φ be given. Suppose (*) holds for less complex formulas.
- $\varphi = \psi_1 \rightarrow \psi_2$. $V_{M_w}(\psi_1 \rightarrow \psi_2, u) = 1$ iff $V_{M_w}(\psi_1) = 0$ or $V_{M_w}(\psi_2) = 1$
iff (IH) $V_M(\psi_1) = 0$ or $V_M(\psi_2)$
iff $V_M(\psi_1 \rightarrow \psi_2) = 0$
- $\varphi = \sim\psi$. Similarly

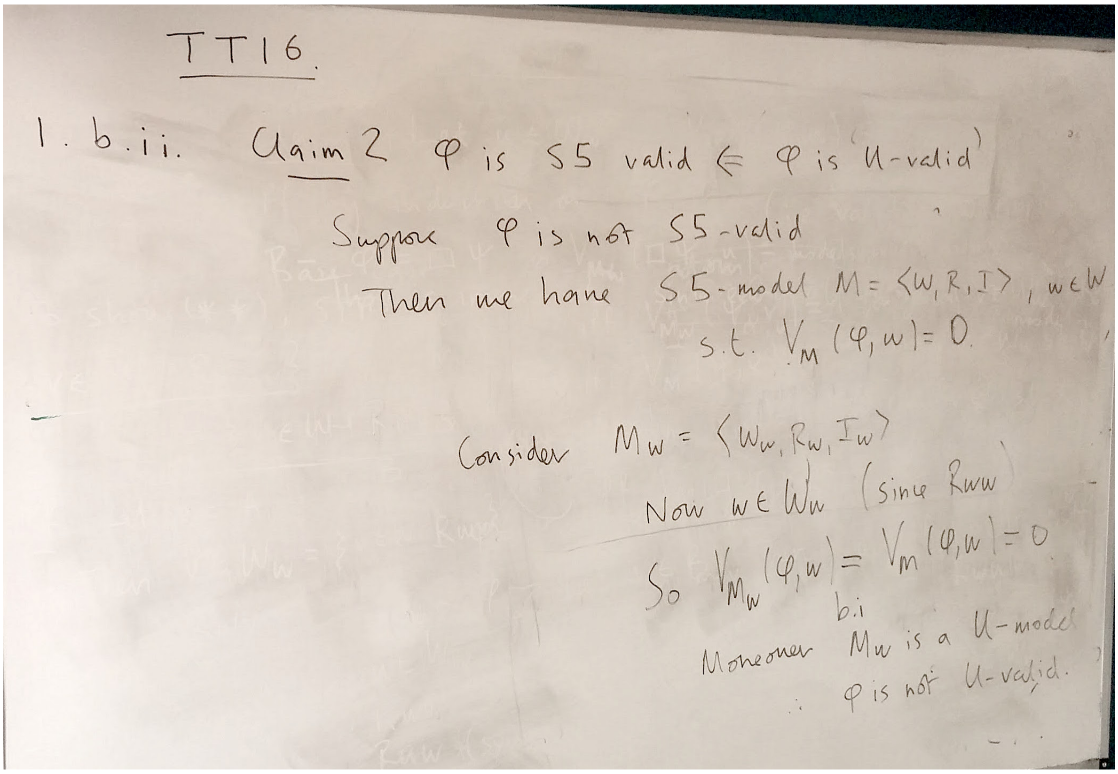
(1.b.i) cont.



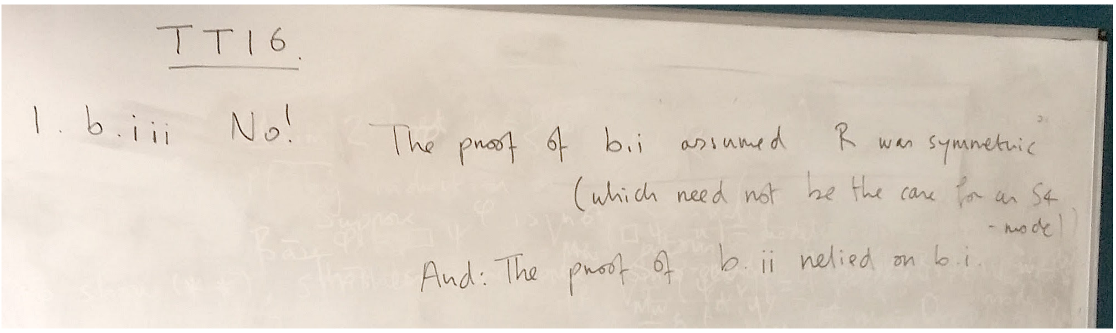
(1.b.ii)



(1.b.ii) cont.

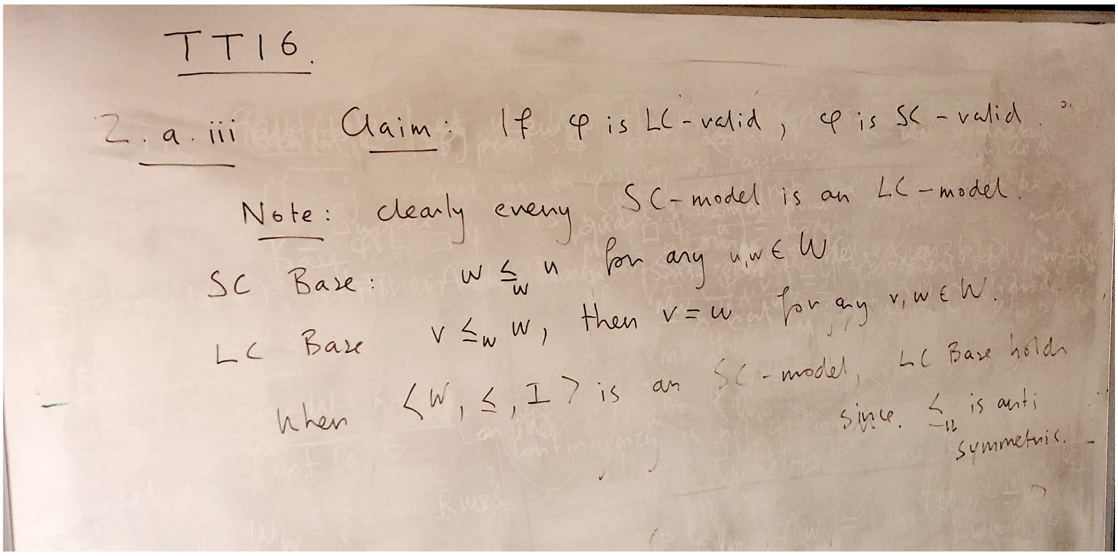


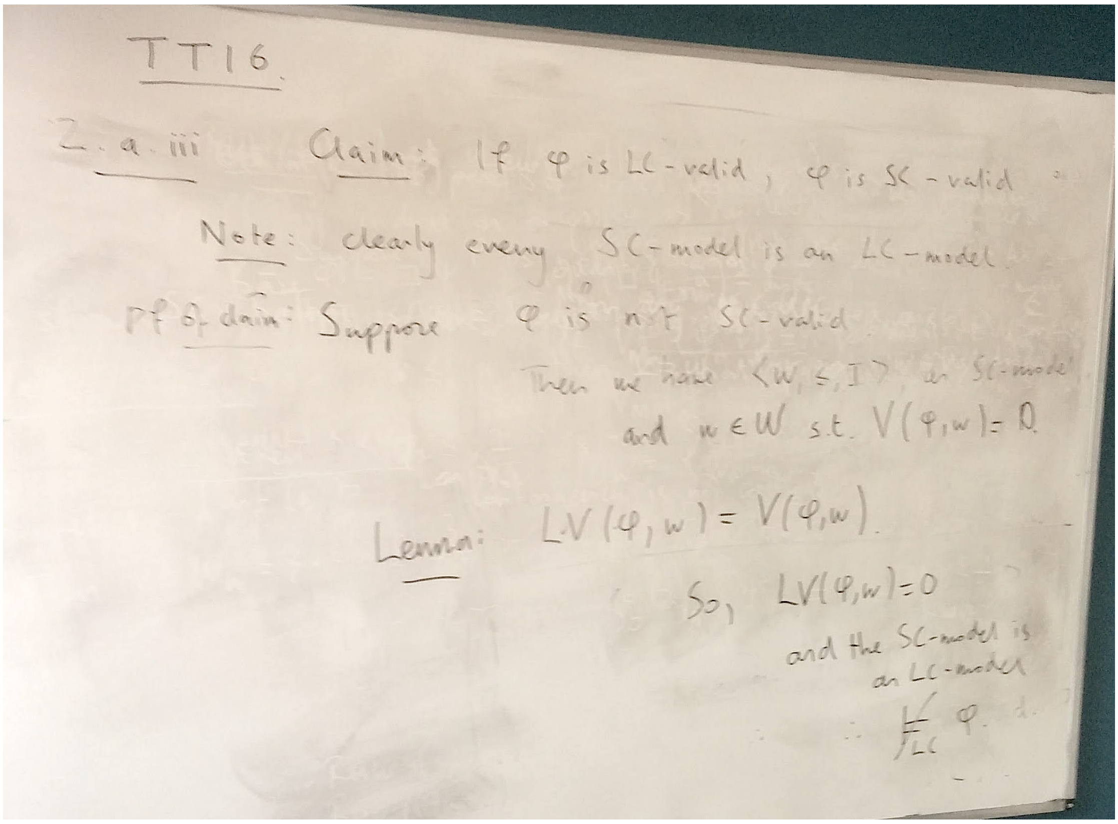
(1.b.iii)



Question 2

(2.a.iii)





TT16. For $\langle W, \leq, I \rangle$ an SC-model.

2.a.iii Lemma: $LV(\varphi, w) = V(\varphi, w)$.

Pf by induction. The base case and cases for $\sim$, $\rightarrow$, and $\Box$ are routine.

That leaves: $\Box\!\!\rightarrow$. RTP: $LV(\varphi \Box\!\!\rightarrow \psi, w) = V(\varphi \Box\!\!\rightarrow \psi, w)$

(assuming $LV(\varphi, u) = V(\varphi, u)$, $LV(\psi, u) = V(\psi, u)$.)

Notn: Suppose there is a 'φ-world' u (i.e $V(\varphi, u) = 1$). Write w_φ for the max. close φ-world to u.

TT16. For $\langle W, \leq, I \rangle$ an SC-model.

2.a.iii Lemma: $LV(\varphi, w) = V(\varphi, w)$.

Pf: Suppose $LV(\varphi \Box\!\!\rightarrow \psi, w) = 1$ (*)

$V(\varphi \Box\!\!\rightarrow \psi, w) = 0$ (†) (for *)

Then there is some φ-world.

Moreover from (*) there is some φ-world u and $LV(\varphi \rightarrow \psi, v) = 1$ for any $v \leq_w u$ (‡)

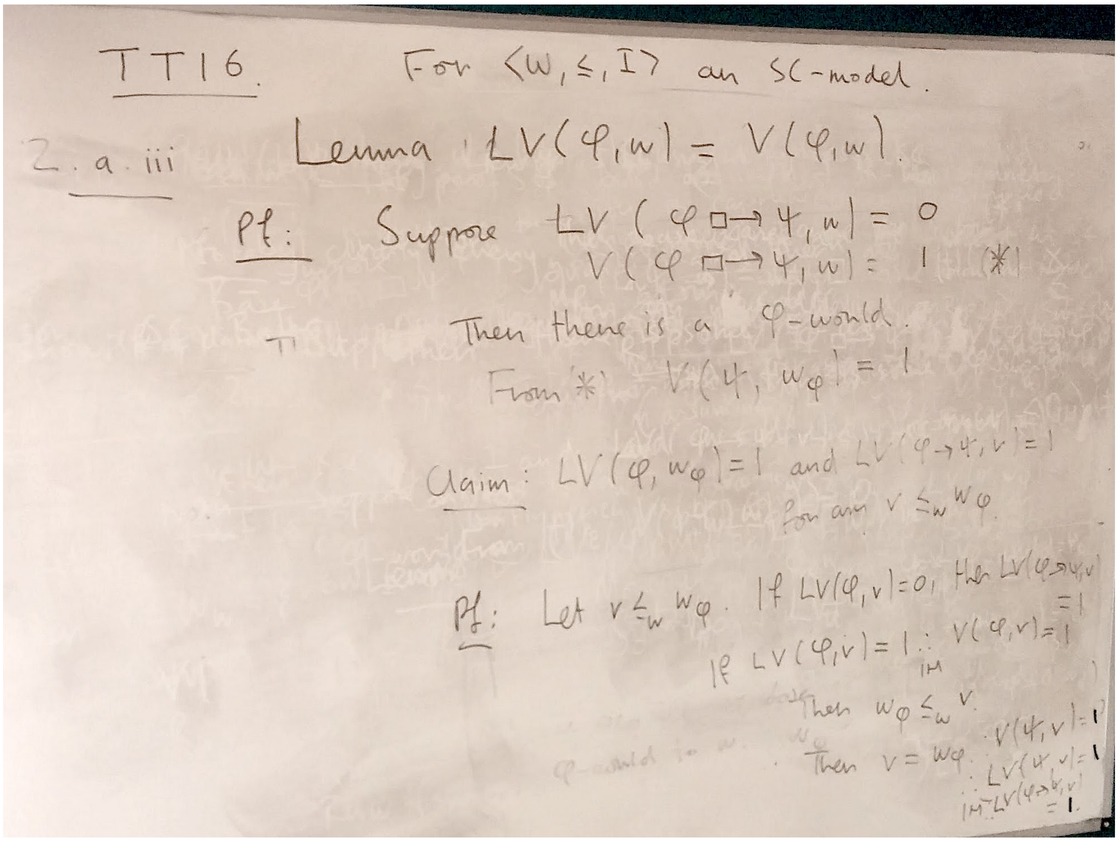
From (†) $V(\psi, w_\varphi) = 0$ $\therefore_{IH}$ $LV(\psi, w_\varphi) = 0$

Moreover $V(\varphi, w_\varphi) = 1$ $\therefore_{IH}$ $LV(\varphi, w_\varphi) = 1$

$\therefore LV(\varphi \rightarrow \psi, w_\varphi) = 0$

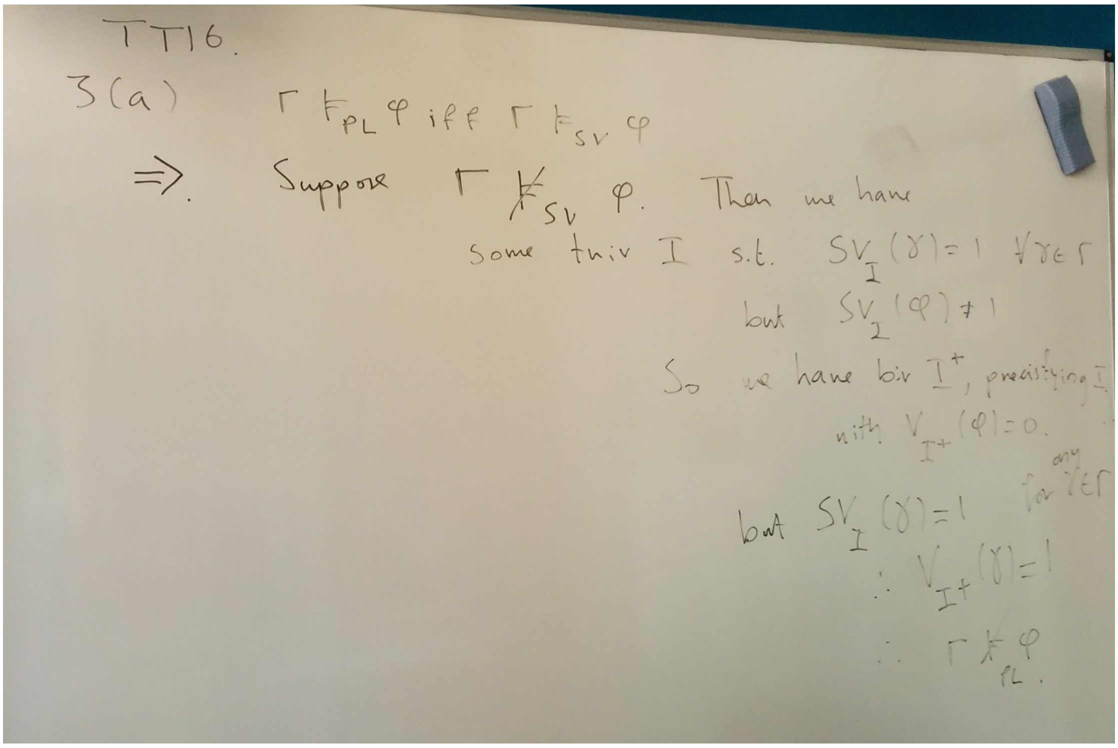
Since w_φ is max close φ-world to w. $\therefore w_\varphi \leq_w u$. $\therefore LV(\varphi \rightarrow \psi, w_\varphi) = 1$.

*

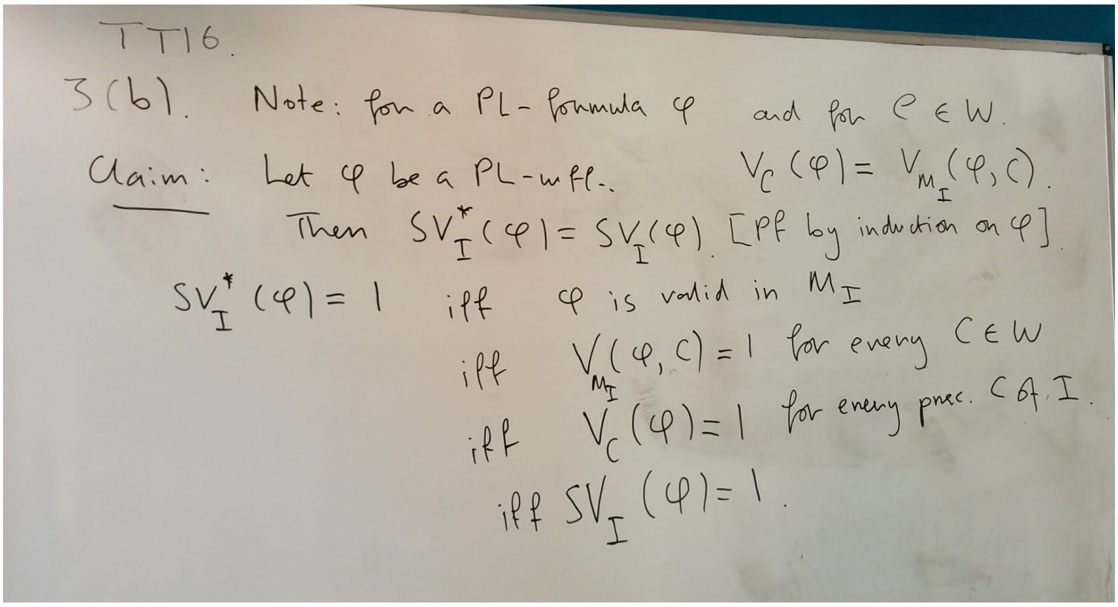

TT16.
For ⟨W, ≤, I⟩ an SC-model.
2.a.iii
Lemma: LV(φ, w) = V(φ, w).
Pf: Suppose LV(φ □→ ψ, w) = 0
V(φ □→ ψ, w) = 1 (*)
Then there is a φ-world.
From (*) V(ψ, wφ) = 1
Claim: LV(φ, wφ) = 1 and LV(φ→ψ, v) = 1
for any v ≤w wφ.
Pf: Let v ≤w wφ. If LV(φ, v) = 0, then LV(φ→ψ, v) = 1
If LV(φ, v) = 1 ∴ V(φ, v) = 1 IH
Then wφ ≤w v.
Then v = wφ. ∴ V(ψ, v) = 1 ∴ LV(ψ, v) = 1
IH ∴ LV(φ→ψ, v) = 1.

Question 3

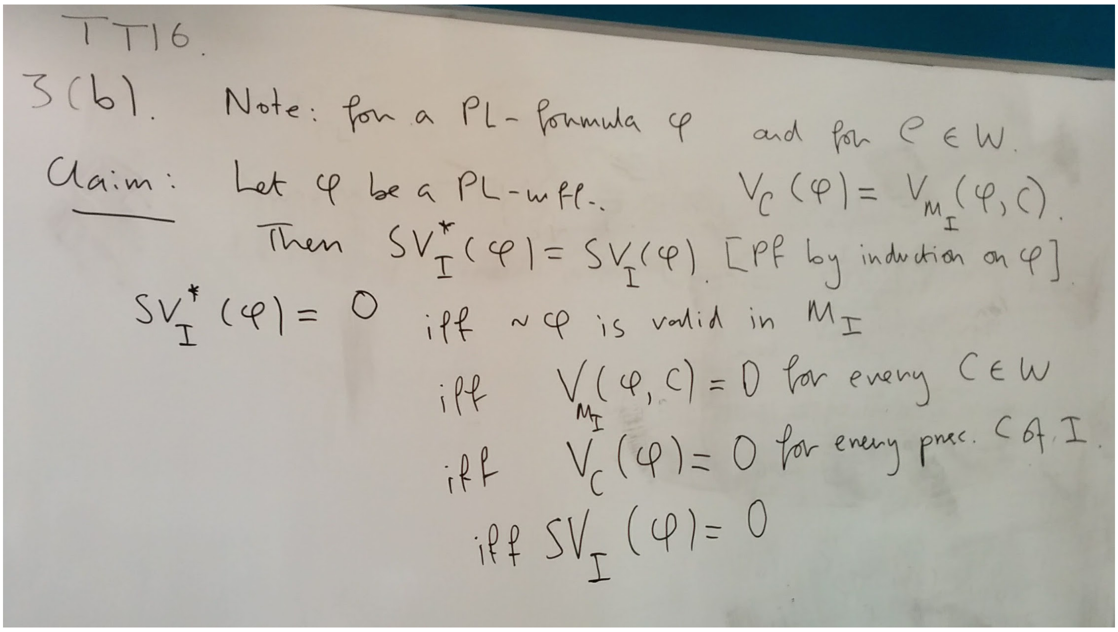
(3.a)



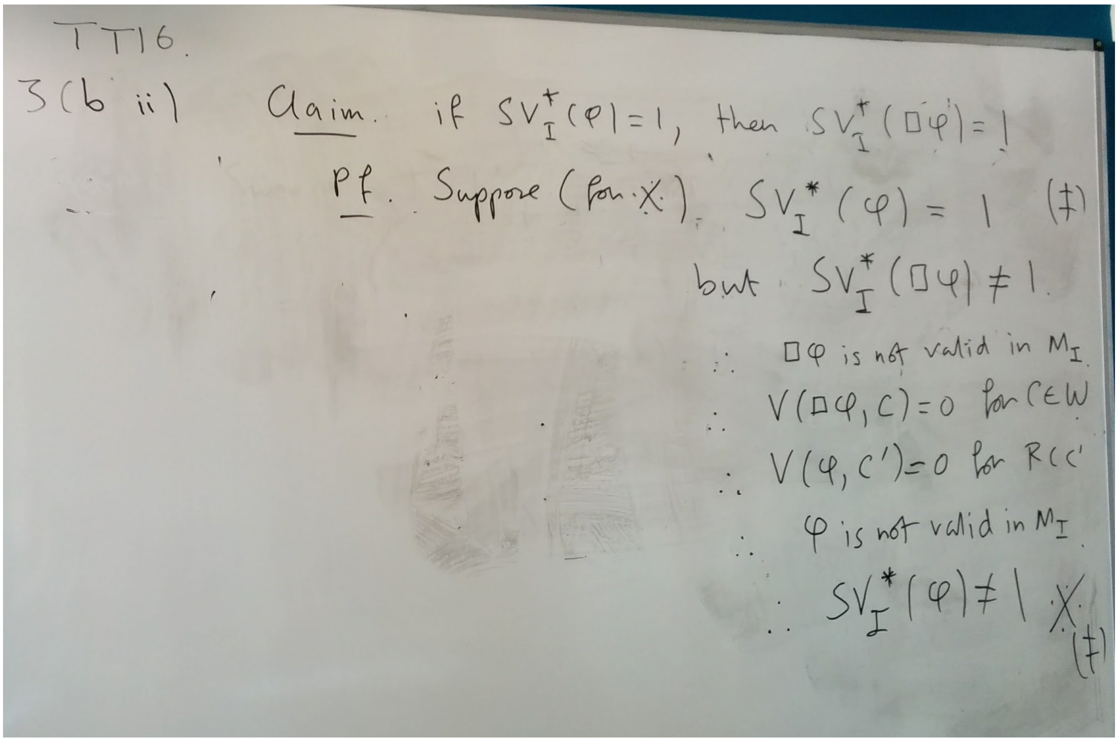
(3.b.i)



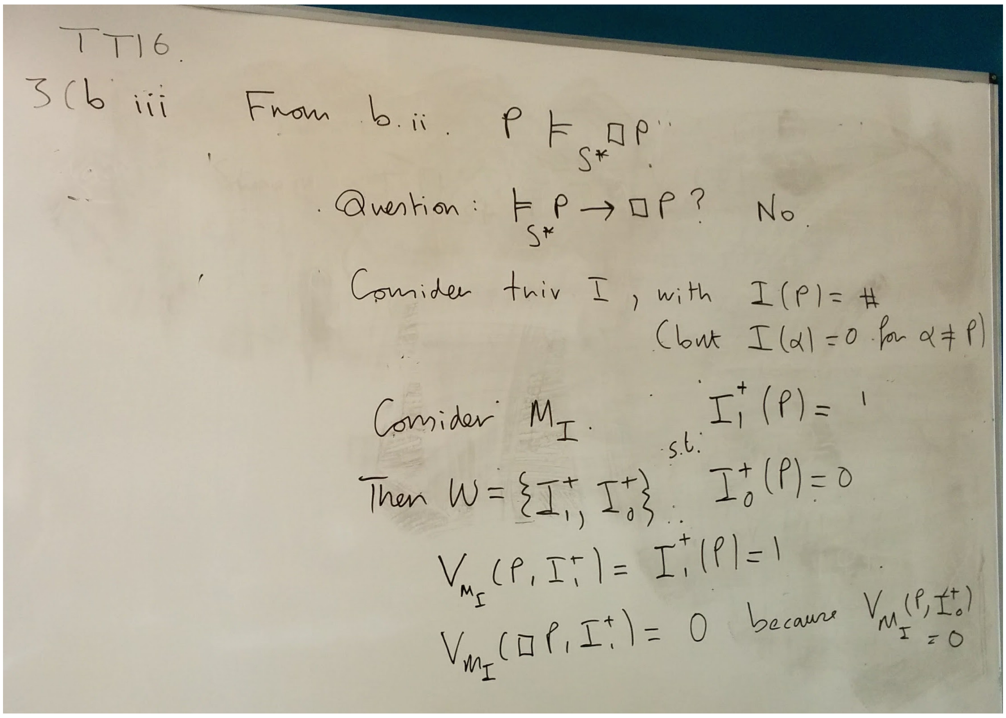
(3.b.i) cont.

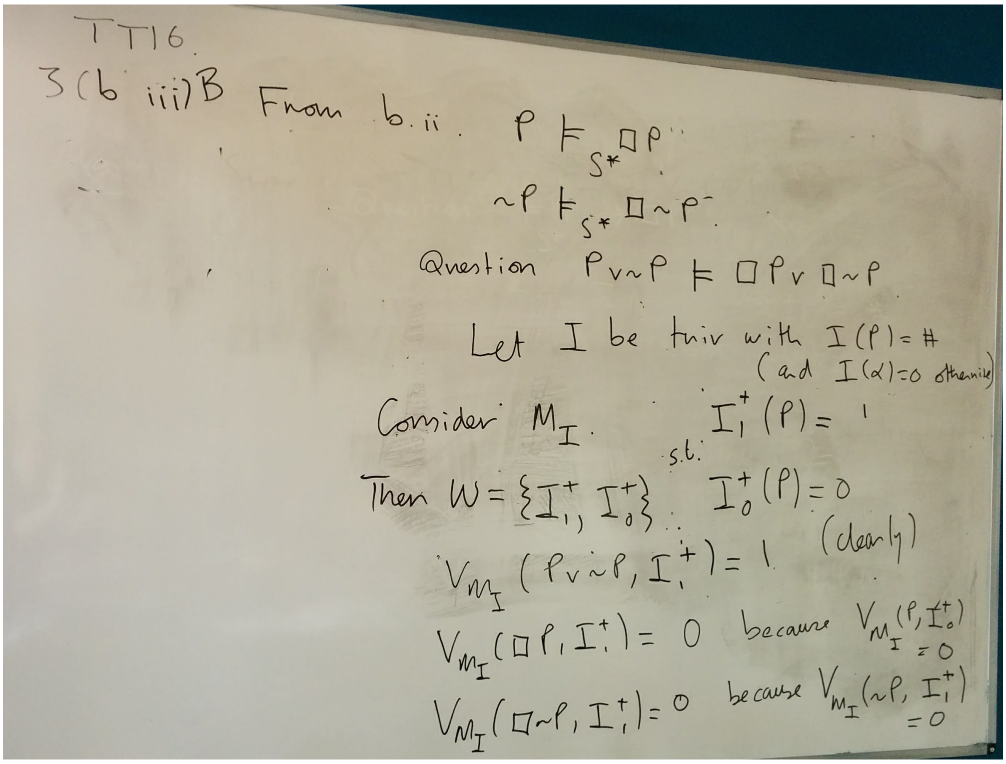


(3.b.ii)



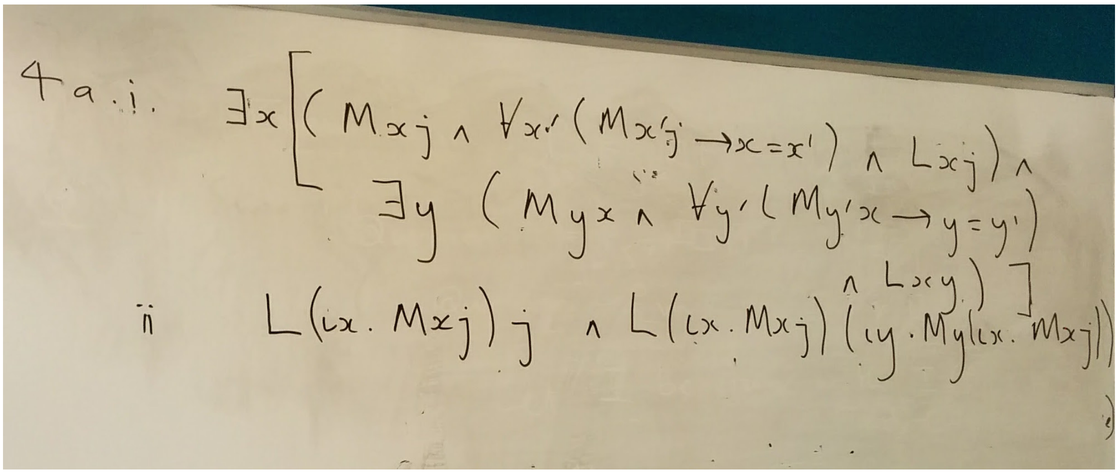
(3.b.iii)


TT16.
3(b iii
From b.ii. p ⊨S* □p
Question: ⊨S* p → □p ? No.
Consider triv I, with I(p) = #
(but I(α) = 0 for α ≠ p)
Consider M_I.
I_1^+(p) = 1
s.t.
I_0^+(p) = 0
Then W = {I_1^+, I_0^+}
V_{M_I}(p, I_1^+) = I_1^+(p) = 1
V_{M_I}(□p, I_1^+) = 0 because V_{M_I}(p, I_0^+) = 0

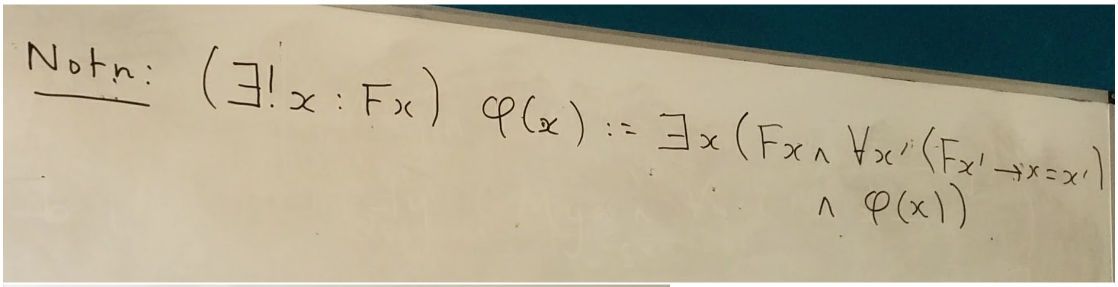

TT16.
3(b iii)B
From b.ii. p ⊨S* □p
~p ⊨S* □~p
Question p∨~p ⊨ □p ∨ □~p
Let I be triv with I(p) = #
(and I(α) = 0 otherwise)
Consider M_I.
I_1^+(p) = 1
s.t.
I_0^+(p) = 0
Then W = {I_1^+, I_0^+}
V_{M_I}(p∨~p, I_1^+) = 1 (clearly)
V_{M_I}(□p, I_1^+) = 0 because V_{M_I}(p, I_0^+) = 0
V_{M_I}(□~p, I_1^+) = 0 because V_{M_I}(~p, I_1^+) = 0

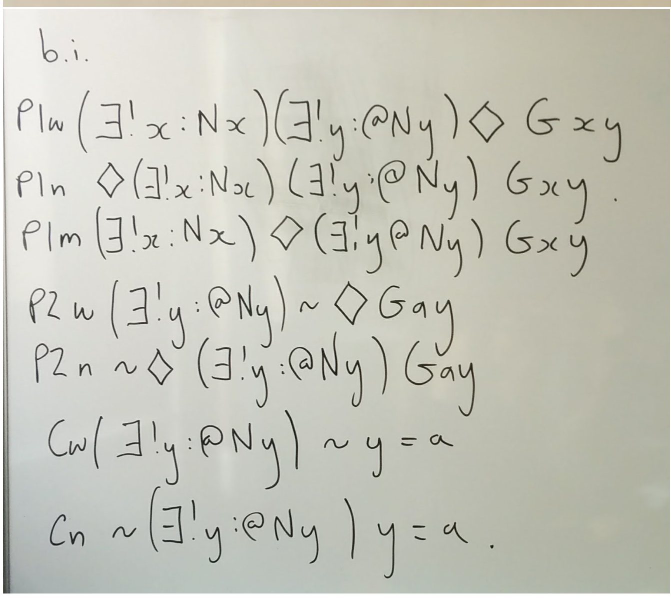
Question 4

(4.a)

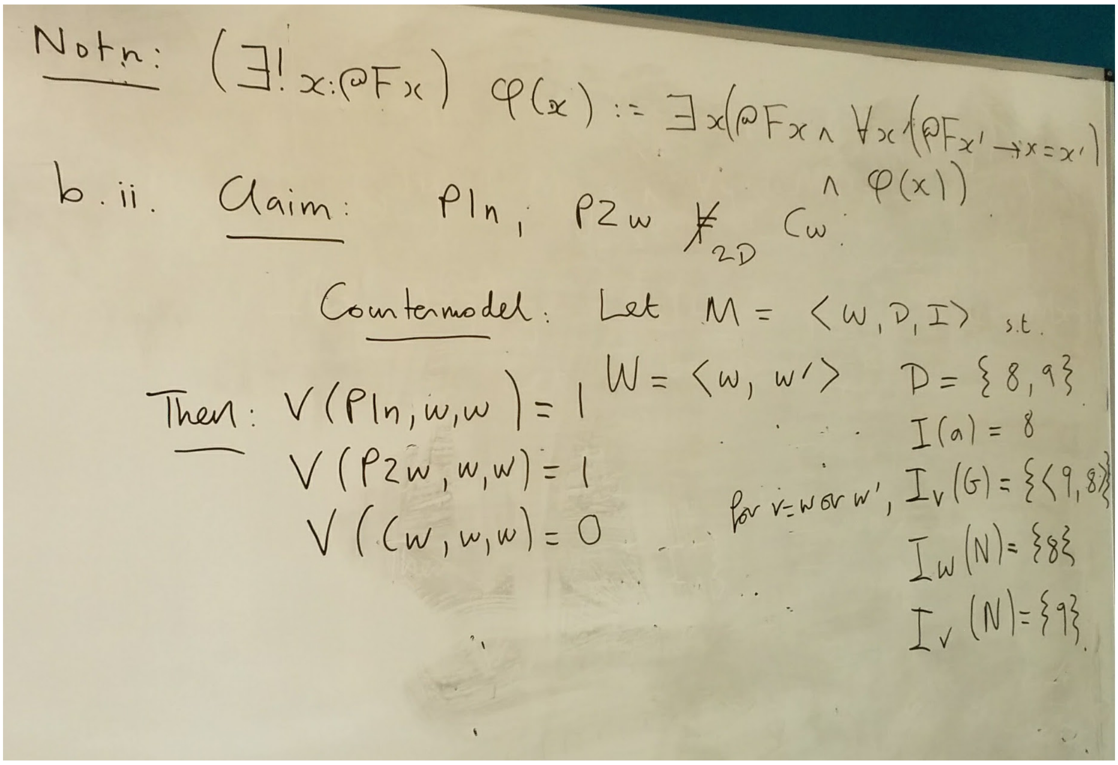


(4.b.i)

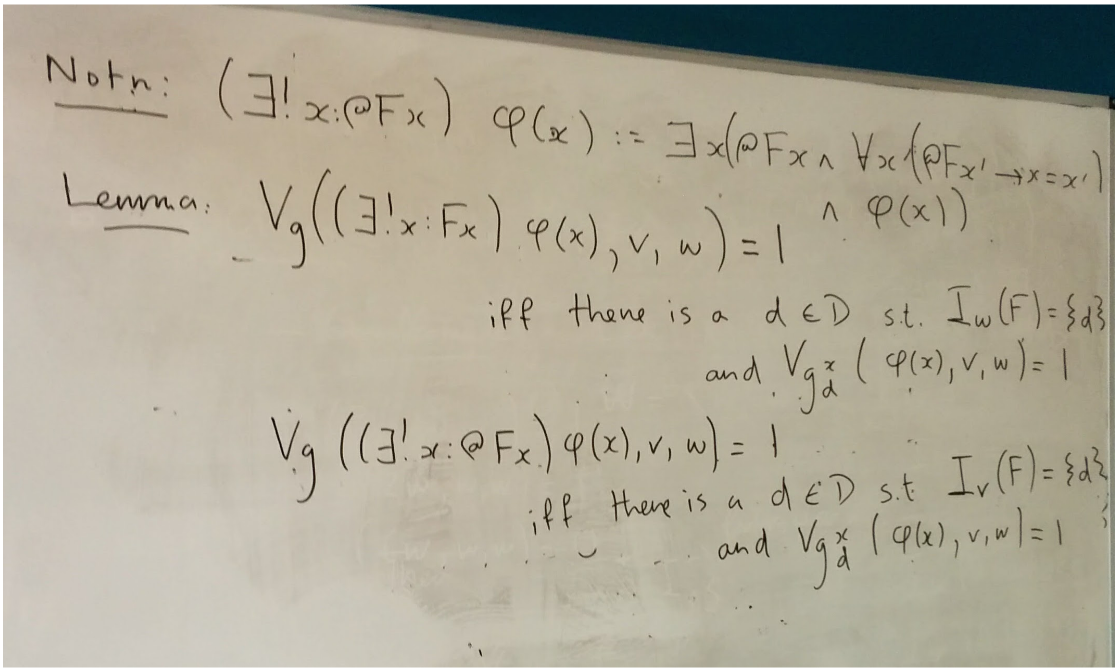




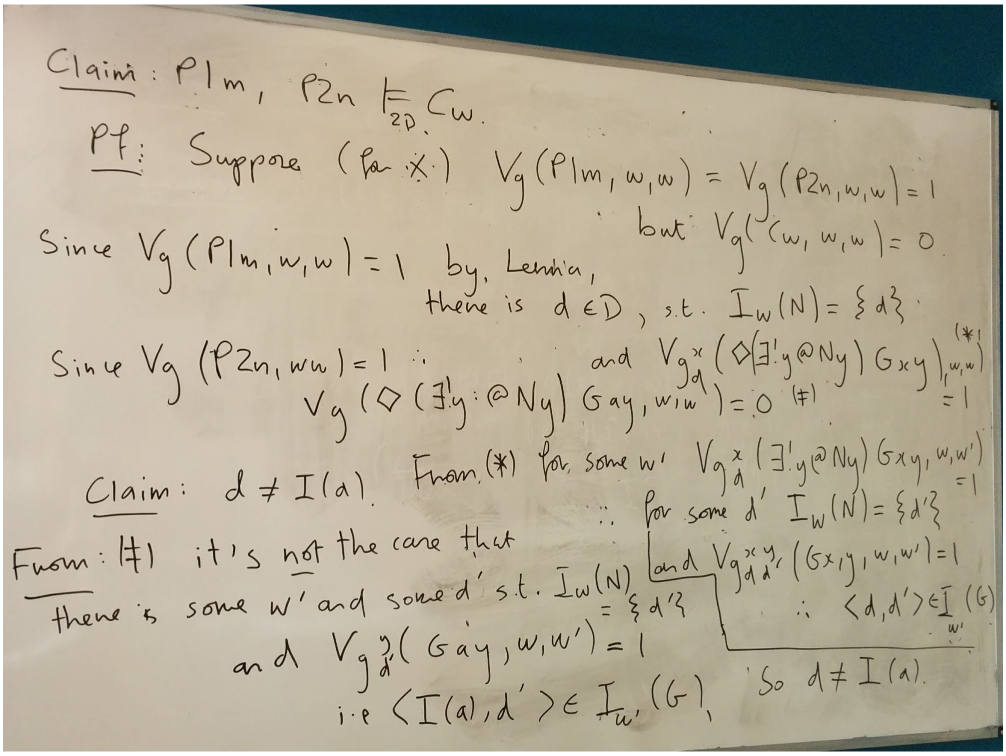
(4.b.ii)

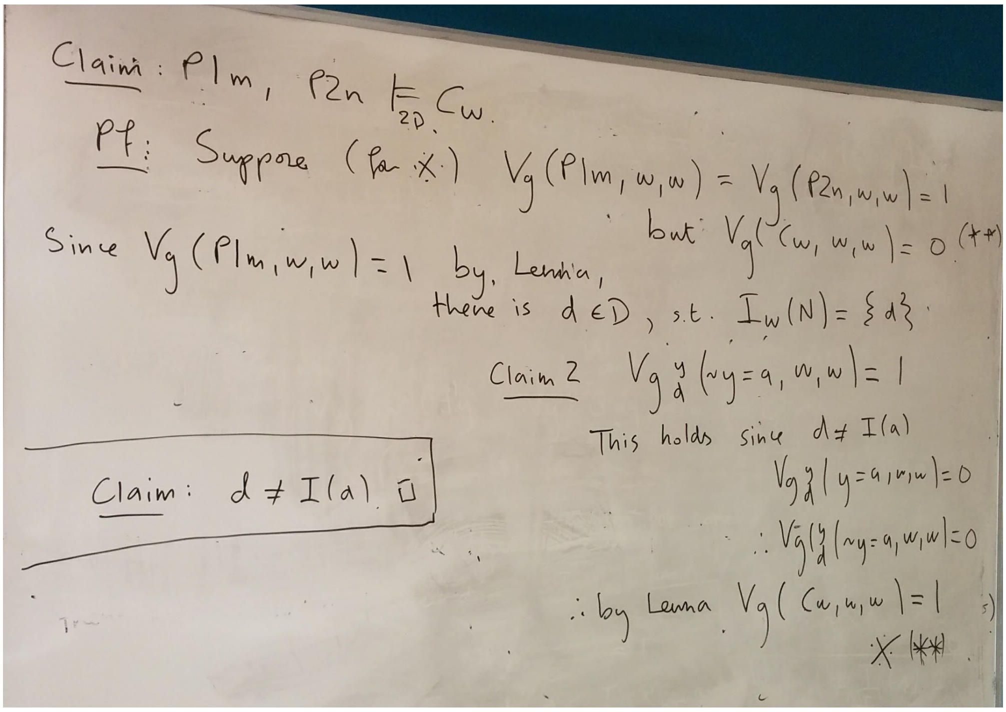

Notn: (∃! x:@Fx) φ(x) := ∃x(@Fx ∧ ∀x'(@Fx' → x=x') ∧ φ(x))
b.ii. Claim: P1n, P2w ⊭2D Cw.
Countermodel: Let M = ⟨W, D, I⟩ s.t.
W = ⟨w, w'⟩ D = {8, 9}
I(a) = 8
for v = w or w', Iv(G) = {⟨9, 8⟩}
Iw(N) = {8}
Iv(N) = {9}.
Then: V(P1n, w, w) = 1
V(P2w, w, w) = 1
V(Cw, w, w) = 0

(4.b.iii)


Notn: (∃! x:@Fx) φ(x) := ∃x(@Fx ∧ ∀x'(@Fx' → x=x') ∧ φ(x))
Lemma: Vg((∃!x:Fx) φ(x), v, w) = 1
iff there is a d ∈ D s.t. Iw(F) = {d}
and Vg x/d (φ(x), v, w) = 1
Vg((∃! x:@Fx) φ(x), v, w) = 1
iff there is a d ∈ D s.t. Iv(F) = {d}
and Vg x/d (φ(x), v, w) = 1

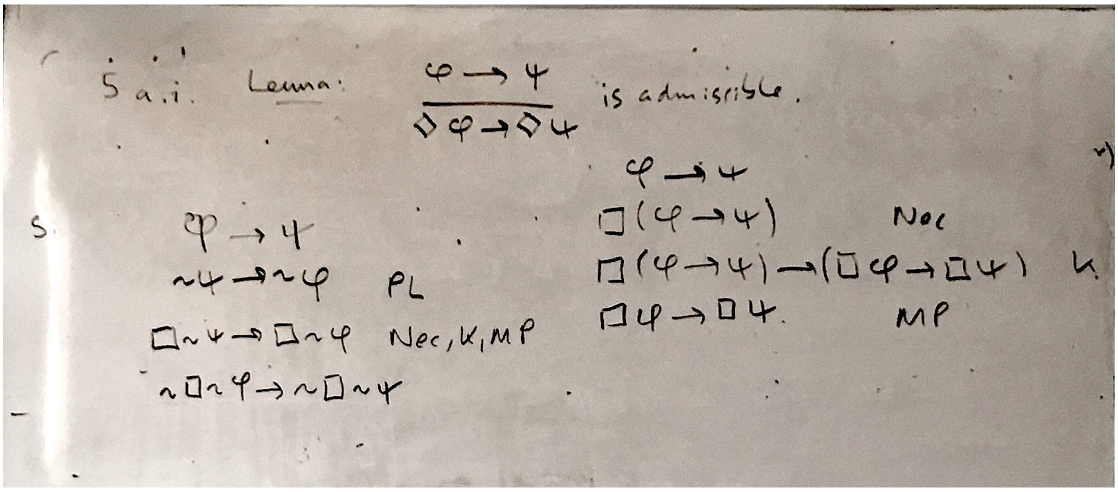
(4.b.iii) cont.


Claim: P1m, P2n ⊨2D Cw.
Pf: Suppose (for X) Vg(P1m, w, w) = Vg(P2n, w, w) = 1
but Vg(Cw, w, w) = 0
Since Vg(P1m, w, w) = 1 by Lemma,
there is d ∈ D, s.t. Iw(N) = {d}
Since Vg(P2n, w, w) = 1
and Vg x d (◇(∃!y@Ny) Gxy), w, w) = 1 (*)
Vg(◇(∃!y:@Ny) Gay, w, w) = 0 (‡)
Claim: d ≠ I(a)
From (*) for some w' Vg x d (∃!y@Ny) Gxy, w, w') = 1
∴ for some d' Iw(N) = {d'}
From: (‡) it's not the case that
there is some w' and some d' s.t. Iw(N) = {d'}
and Vg y d' (Gay, w, w') = 1
i.e. ⟨I(a), d'⟩ ∈ Iw'(G)
and Vg x y d d' (Gxy, w, w') = 1
∴ ⟨d, d'⟩ ∈ Iw'(G)
So d ≠ I(a).


Claim: P1m, P2n ⊨2D Cw.
Pf: Suppose (for X) Vg(P1m, w, w) = Vg(P2n, w, w) = 1
but Vg(Cw, w, w) = 0 (**)
Since Vg(P1m, w, w) = 1 by Lemma,
there is d ∈ D, s.t. Iw(N) = {d}
Claim 2 Vg y d (~y=a, w, w) = 1
This holds since d ≠ I(a)
Vg y d (y=a, w, w) = 0
∴ Vg y d (~y=a, w, w) = 0
Claim: d ≠ I(a) □
∴ by Lemma Vg(Cw, w, w) = 1
X (**)

Question 5

(5.a.i)



5 a.i. Lemma: $\dfrac{\varphi \to \psi}{\Diamond\varphi \to \Diamond\psi}$ is admissible.

$\varphi \to \psi$

$\Box(\varphi \to \psi)$ Nec

$\Box(\varphi \to \psi) \to (\Box\varphi \to \Box\psi)$ K

$\Box\varphi \to \Box\psi$ MP

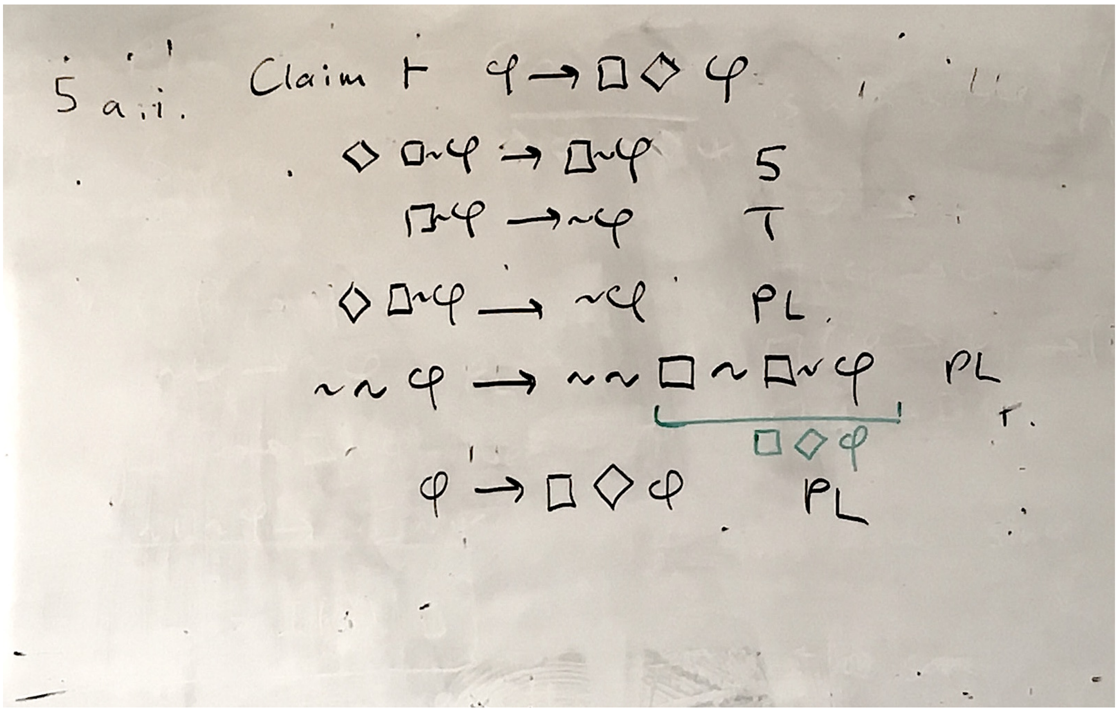
$\varphi \to \psi$

$\sim\psi \to \sim\varphi$ PL

$\Box\sim\psi \to \Box\sim\varphi$ Nec, K, MP

$\sim\Box\sim\varphi \to \sim\Box\sim\psi$

(5.a.ii)



5 a.ii. Claim $\vdash \varphi \to \Box\Diamond\varphi$.

$\Diamond\Box\sim\varphi \to \Box\sim\varphi$ 5

$\Box\sim\varphi \to \sim\varphi$ T

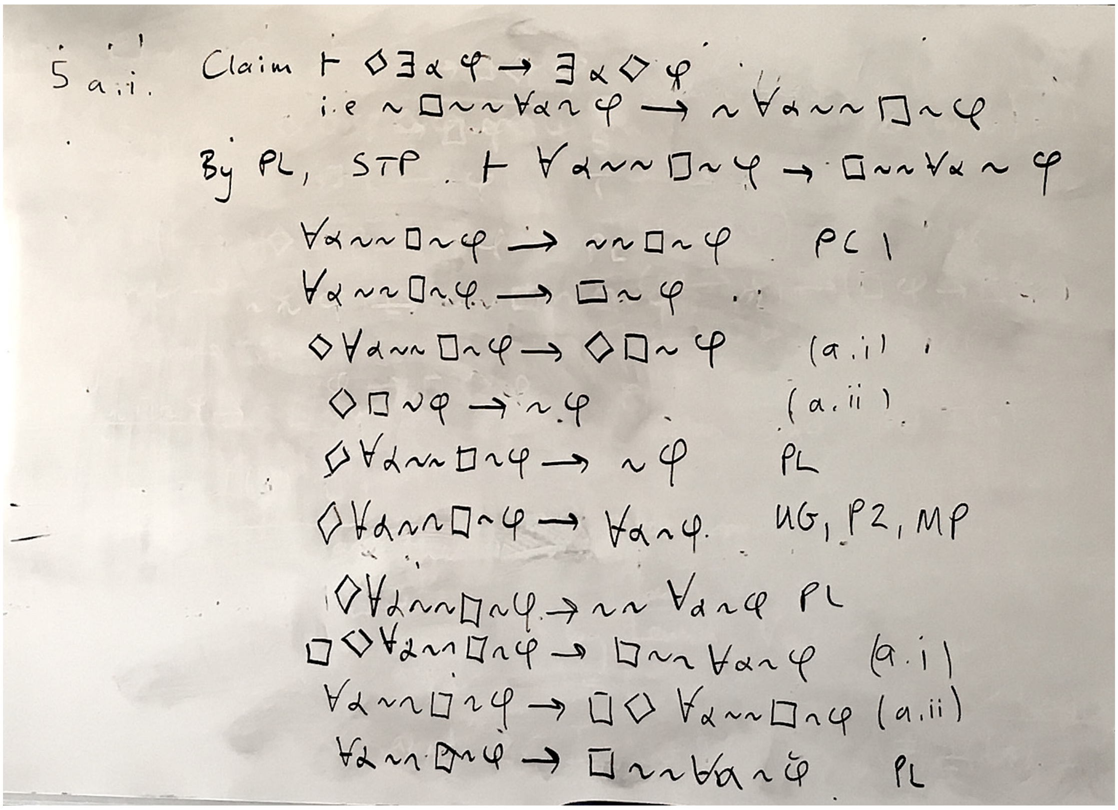
$\Diamond\Box\sim\varphi \to \sim\varphi$ PL

$\sim\sim\varphi \to \sim\sim\Box\sim\Box\sim\varphi$ PL

$\Box\Diamond\varphi$

$\varphi \to \Box\Diamond\varphi$ PL

(5.a.iii)



5 a.i. Claim $\vdash \Diamond \exists \alpha \varphi \rightarrow \exists \alpha \Diamond \varphi$

i.e. $\sim \Box \sim\sim \forall \alpha \sim \varphi \rightarrow \sim \forall \alpha \sim\sim \Box \sim \varphi$

By PL, STP $\vdash \forall \alpha \sim\sim \Box \sim \varphi \rightarrow \Box \sim\sim \forall \alpha \sim \varphi$

$\forall \alpha \sim\sim \Box \sim \varphi \rightarrow \sim\sim \Box \sim \varphi$ PC1

$\forall \alpha \sim\sim \Box \sim \varphi \rightarrow \Box \sim \varphi$

$\Diamond \forall \alpha \sim\sim \Box \sim \varphi \rightarrow \Diamond \Box \sim \varphi$ (a.i)

$\Diamond \Box \sim \varphi \rightarrow \sim \varphi$ (a.ii)

$\Diamond \forall \alpha \sim\sim \Box \sim \varphi \rightarrow \sim \varphi$ PL

$\Diamond \forall \alpha \sim\sim \Box \sim \varphi \rightarrow \forall \alpha \sim \varphi$ UG, P2, MP

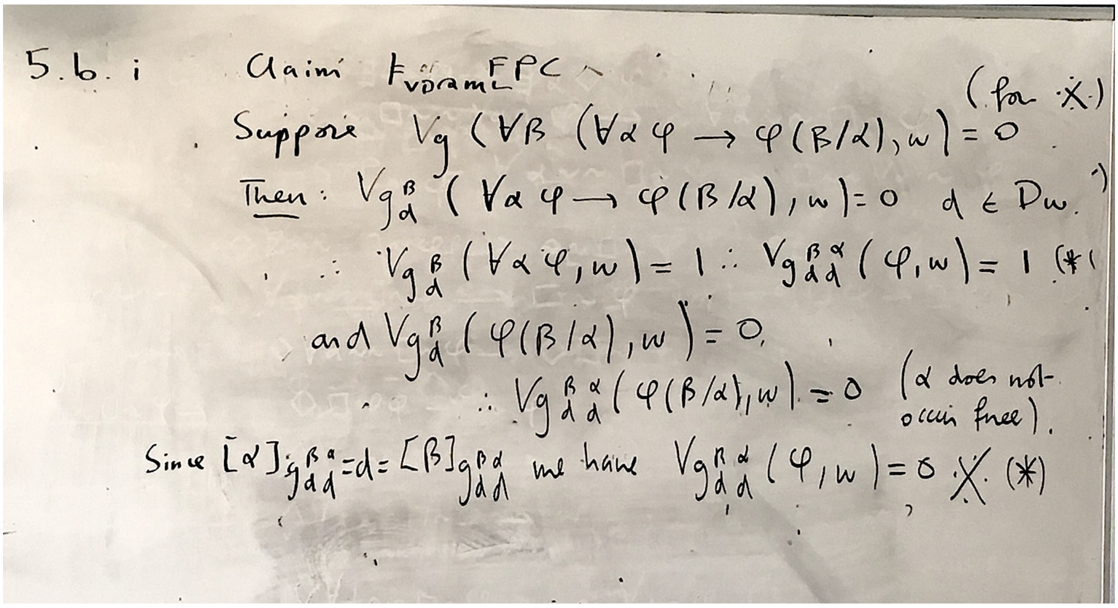
$\Diamond \forall \alpha \sim\sim \Box \sim \varphi \rightarrow \sim\sim \forall \alpha \sim \varphi$ PL

$\Box \Diamond \forall \alpha \sim\sim \Box \sim \varphi \rightarrow \Box \sim\sim \forall \alpha \sim \varphi$ (a.i)

$\forall \alpha \sim\sim \Box \sim \varphi \rightarrow \Box \Diamond \forall \alpha \sim\sim \Box \sim \varphi$ (a.ii)

$\forall \alpha \sim\sim \Box \sim \varphi \rightarrow \Box \sim\sim \forall \alpha \sim \varphi$ PL

(5.b.i)



5.b.i Claim $\vDash_{VDRAML}$ FPC (for X)

Suppose $V_g(\forall \beta (\forall \alpha \varphi \rightarrow \varphi(\beta/\alpha)), w) = 0$

Then: $V_{g^{\beta}_{d}}(\forall \alpha \varphi \rightarrow \varphi(\beta/\alpha), w) = 0$ $d \in D_w$

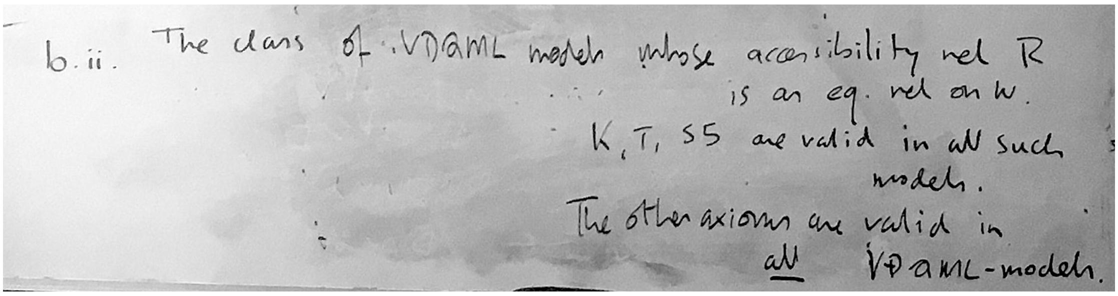
$\therefore V_{g^{\beta}_{d}}(\forall \alpha \varphi, w) = 1 \therefore V_{g^{\beta\alpha}_{dd}}(\varphi, w) = 1$ (*)

and $V_{g^{\beta}_{d}}(\varphi(\beta/\alpha), w) = 0$

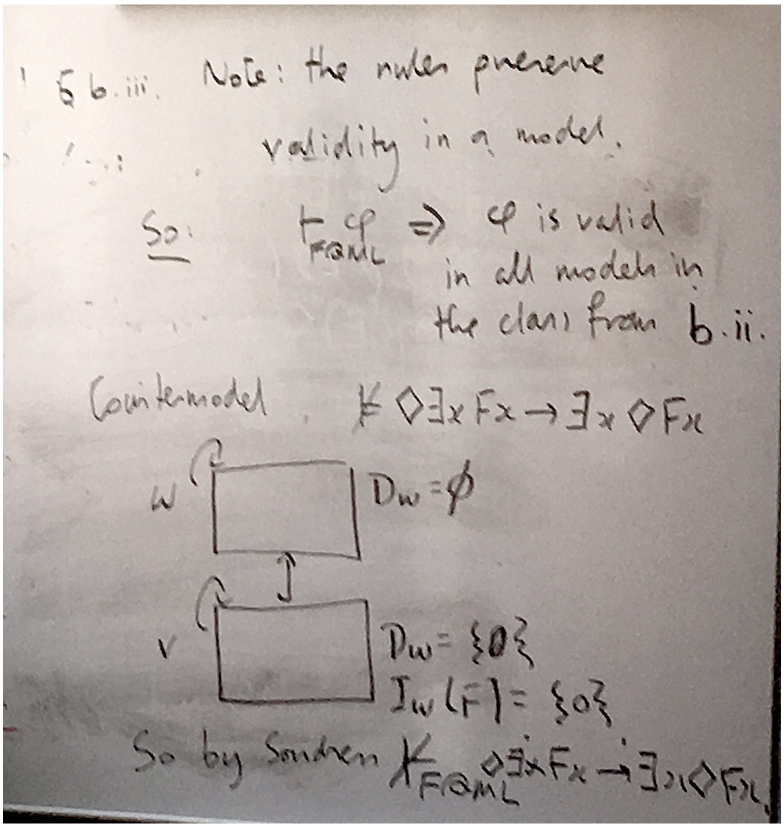
$\therefore V_{g^{\beta\alpha}_{dd}}(\varphi(\beta/\alpha), w) = 0$ (α does not occur free).

Since $[\alpha]_{g^{\beta\alpha}_{dd}} = d = [\beta]_{g^{\beta\alpha}_{dd}}$ we have $V_{g^{\beta\alpha}_{dd}}(\varphi, w) = 0$ X (*)

(5.b.ii)


b.ii. The class of VD@ML models whose accessibility rel R
is an eq. rel on W.
K, T, S5 are valid in all such
models.
The other axioms are valid in
all VD@ML-models.

(5.b.iii)


5b.iii. Note: the rules preserve
validity in a model.
So: ⊢FQML φ ⇒ φ is valid
in all models in
the class from b.ii.
Countermodel ⊭ ◇∃x Fx → ∃x ◇Fx
w
Dw = ∅
v
Dw = {0}
Iw(F) = {0}
So by Soundness ⊬FQML ◇∃xFx → ∃x◇Fx