

AG lit. review questions: week 8

The schematic argument

(Q1) An instance of the schematic argument may be stated as follows:

Sets₀ get **Collected**₁. Any zero or more sets₀ are collected₁.

$$\forall ss_0 \exists s_1 (s_1 \equiv_1 ss_0)$$

Urelements₀ remain **Urelements**₁. Every urelement₀ is an urelement₁.

$$\forall x_0 (\neg \beta_0 x_0 \rightarrow \neg \beta_1 x_0)$$

No Comprehensive₁ **Domain**₀. No zero or more items₀ comprise every item₁.

$$\neg \exists x x_0 \forall x_1 (x_1 < xx_0)$$

Is the argument valid? What motivates its premisses?

(Q2) Why is a sorted argument required?

(Q3) I write:

What should the absolutist make of the argument? Assuming he does not object to reasoning in PFO_{0,1}, he must either accept the conclusion or reject a premiss. The former option is, strictly speaking, compatible with absolutism. After all, No Comprehensive₁ Domain₀ only denies the comprehensiveness of subdomains of M_0 . The absolutist may consequently go back on his initial claim that M_0 is absolutely comprehensive without renouncing absolutism.

The dialectical position this leaves him in, however, is clearly untenable. (EMoL, 183)

Is it?

(Q4) Can we make sense of the schematic generalization of this argument? What is its side-condition?

The modal argument

(Q5) What (in my terminology) is the difference between a plurality being *collected* and being *collectable*? Or a condition being (plurally-) *comprehended* or *comprehensible*?

(Q6) The modal argument may be stated as follows:

Unlimited Collectability for Sets. Absolutely any zero or more sets are collectable.

$$\square \forall xx(\beta xx \rightarrow \diamond \exists s(s \equiv^\diamond xx))$$

No Absolutely Comprehensive Domain. Absolutely no zero or more items comprise absolutely everything.

$$\neg \diamond \exists xx \square \forall x(x <^\diamond xx)$$

(In the argument: $\beta xx =_{\text{df}} \forall x(x < xx \rightarrow \beta x)$)

(Q7) Is the premiss coherent? Is it consonant with standard set theory?

(Q8) Does this argument rely on an analogue of Urelements_i remain Urelements_j?

Actualism and Potentialism

(Q9) How might we best understand the difference between actualism and potentialism? How do these views relate to so called Cantorian absolutism and Zermelian relativism?

(Q10) Does the difference between actualism and potentialism surface at the level of their formal set theory (e.g. ZFCSU_p)?

(Q11) Is Zermelo right to claim that the potentialist secures ‘unlimited applicability’ for set theory and resolves the paradoxes without ‘constriction and mutilation’? (1930, pp. 427, 431)

The why-question/explanatory challenge

(Q12) ‘What makes uncollectable pluralities uncollectable?’ (EMoL, 205) ‘Why is the universe of sets not a set? (Soysal, p. 1) Are we asking the same question?

(Q13) What is the Minimal Explanation to the why-question? Is it somehow deficient (i) as an answer to Soysal’s ‘why-question’ (ii) to my ‘explanatory challenge’?

(Q14) What is Soysal’s response to the ‘arbitrary threshold objection’? Is it successful?

(Q15) Soysal writes:

[To answer the why question for the cumulative hierarchy] the potentialist will have to provide us with a version of the minimal explanation from within modal set theory.

This isn't yet a devastating problem for the potentialist. Indeed, potentialists may reply that modal set theory captures some deeper truths about sets than does ZFC, and hence claim that the minimal explanation within modal set theory is more satisfactory than the minimal explanation within ZFC. But this is where the potentialist's primitive and idiosyncratic notion of modality causes trouble. The second step of the argument is thus to note, as we did above in Sect. 4.1, that the potential and iterative hierarchies are isomorphic, and modal and non-modal set theories are mutually interpretable. This means we cannot get any grip on the potentialist's modality by merely considering the set of true sentences containing ' \Box ' and ' \Diamond '. If, moreover, we are given no independent grip on potentialist's notion of modality (because we are told it is primitive and idiosyncratic to set theory), then what stops us from simply interpreting the domains of the worlds w_α as stages V_α defined in ZFC? What exactly is added by the ' \Box ' and ' \Diamond ' in front of quantifiers? Potentialism on this option starts to look like a notational variant of set theory. And this surely affects its explanatory power: To say that the universe of sets is not a set because it is "potential" in that at any stage, we "can" form more sets in this unspecified and idiosyncratic sense of "can" is not far from giving a dormitive virtue explanation, or saying nothing at all. In other words, simply having unexplained 'P' and 'Q' in front of the quantifiers in the minimal explanation doesn't make the potentialist explanation any deeper or more informative than the minimal explanation. The potentialist explanation with this unexplained, primitive and idiosyncratic notion of modality doesn't provide any deeper insight on the why-question than the minimal explanation. (p. 14)

How might a potentialist respond?

(Q16) What is the Conception-based Explanation? Does it improve on its actualist and potentialist competitors?

Absolutist-friendly indefinite extensibility

(Q17) Williamson writes: "For given any reasonable assignment of meaning to the word 'set' we can assign it a more inclusive meaning while feeling that we are going on in the same way... the inconsistency is not in any one meaning...it is in the attempt to combine all the different meanings that we could reasonably assign it into a single super-meaning." (1998a, p. 20)

This suggests instead denying $U\text{relements}_i$ remain $U\text{relements}_j$. Does this provide an attractive absolutist response to the argument?