# INTRODUCTION TO LOGIC

## Lecture 1

Validity

Introduction to Sets and Relations.

James Studd

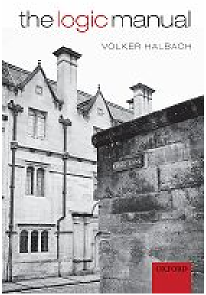
Pure logic is the ruin of the spirit.

*Antoine de Saint-Exupéry*

# Outline

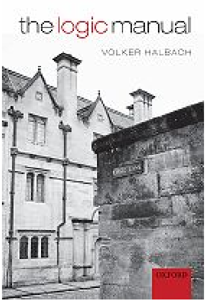
# Resources

- The Logic Manual



# Resources

- The Logic Manual
- **logicmanual.philosophy.ox.ac.uk**



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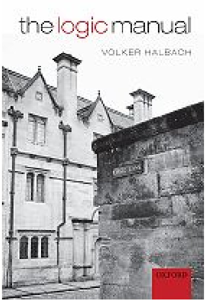
  - Exercises booklet
  - Lecture slides
  - Worked examples
  - Past examination papers some with solutions



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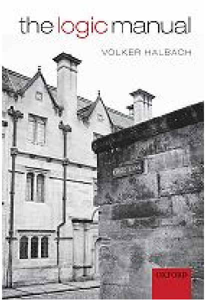


- Mark Sainsbury: *Logical Forms: An Introduction to Philosophical Logic*, Blackwell, second edition, 2001, chs. 1–2.

# Why logic?

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- Logic allows us to rigorously test validity.
- Modern philosophy assumes familiarity with logic.
- Used in linguistics, mathematics, computer science,...
- Helps us make fine-grained conceptual distinctions.
- Logic is compulsory.

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**First approximation.**

When an argument is valid, the truth of the premisses **guarantees** the truth of the conclusion. 10

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- Validity does **not** depend on contingent facts.
- Validity does **not** depend on laws of nature.
- Validity does **not** depend on the meanings of subject-specific expressions.
- Validity depends purely on the 'form' of the argument.

# Examples

**Argument 1**

Zeno is a tortoise.
Therefore, Zeno is toothless.

# Examples

**Argument 1** **Not valid**

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The truth of the premiss does not provide a sufficiently strong guarantee of the truth of the conclusion

# Examples

**Argument 1** **Not valid**

Zeno is a tortoise.
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The truth of the premiss does not provide a sufficiently strong guarantee of the truth of the conclusion

**Argument 2**

Zeno is a tortoise.
All tortoises are toothless.
Therefore, Zeno is toothless.

# Examples

**Argument 1** **Not valid**

Zeno is a tortoise.
Therefore, Zeno is toothless.

The truth of the premiss does not provide a sufficiently strong guarantee of the truth of the conclusion

**Argument 2** **Valid**

Zeno is a tortoise.
All tortoises are toothless.
Therefore, Zeno is toothless.

# Validity 2/3

## Characterisation (p. 19)

An argument is **logically valid** if and only if:
there is no interpretation under which:

**(i)** the premisses are all true, and

**(ii)** the conclusion is false.

# Argument 1 revisited

**Argument 1**

Zeno is a tortoise.

Therefore, Zeno is toothless.

# Argument 1 revisited

**Argument 1** **Not valid**

Zeno is a tortoise.
Therefore, Zeno is toothless.

There is an interpretation under which:

**(i)** the premiss is true, and
**(ii)** the conclusion is false.

# Argument 1 revisited

**Argument 1** **Not valid**

Zeno is a tortoise.
Therefore, Zeno is toothless.

**Argument 1a**

Boris Johnson is a Conservative.
Therefore, Boris Johnson is a Liberal Democrat.

There is an interpretation under which:

**(i)** the premiss is true, and
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# Argument 1 revisited

**Argument 1** **Not valid**

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There is an interpretation under which:

**(i)** the premiss is true, and
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# Argument 2 revisited

**Argument 2** **Valid**

Zeno is a tortoise.
All tortoises are toothless.
Therefore, Zeno is toothless.

# Argument 2 revisited

**Argument 2** **Valid**

Zeno is a tortoise.
All tortoises are toothless.
Therefore, Zeno is toothless.

**Argument 2a**

Boris Johnson is a Conservative.
All Conservatives are Liberal Democrats.
Therefore, Boris Johnson is a Liberal Democrat.

# Argument 2 revisited

**Argument 2** **Valid**

Zeno is a tortoise.
All tortoises are toothless.
Therefore, Zeno is toothless.

**Argument 2a** **Valid**

Boris Johnson is a Conservative.
All Conservatives are Liberal Democrats.
Therefore, Boris Johnson is a Liberal Democrat.

# Argument 2 revisited

**Argument 2** **Valid**

Zeno is a tortoise.
All tortoises are toothless.
Therefore, Zeno is toothless.

**Argument 2a** **Valid**

Boris Johnson is a Conservative.
All Conservatives are Liberal Democrats.
Therefore, Boris Johnson is a Liberal Democrat.

**Argument 2b**

Radon is a noble gas.
All noble gases are chemical elements.
Therefore, Radon is a chemical element.

# Argument 2 revisited

**Argument 2** **Valid**

Zeno is a tortoise.
All tortoises are toothless.
Therefore, Zeno is toothless.

**Argument 2a** **Valid**

Boris Johnson is a Conservative.
All Conservatives are Liberal Democrats.
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**Argument 2b** **Valid**

Radon is a noble gas.
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# Argument 2 revisited

**Argument 2** **Valid**

Zeno is a tortoise.
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Therefore, Zeno is toothless.

**Argument 2a** **Valid**

Boris Johnson is a Conservative.
All Conservatives are Liberal Democrats.
Therefore, Boris Johnson is a Liberal Democrat.

**Argument 2b** **Valid**

Radon is a noble gas.
All noble gases are chemical elements.
Therefore, Radon is a chemical element.

Note: argument 2a is a valid argument with a false conclusion.

# Validity 3/3.

**Characterisation (p. 19)**

An argument is **logically valid** if and only if:
there is no [uniform] interpretation [of subject-specific expressions] under which:

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- Each occurrence of an expression interpreted in the same way
- Logical expression keep their usual English meanings.

# Subject-specific versus logical expressions

**Examples: logical terms**

all, every, some, no.
not, and, or, unless, if, only if, if and only if.

# Subject-specific versus logical expressions

**Examples: logical terms**

all, every, some, no.
not, and, or, unless, if, only if, if and only if.

**Examples: subject-specific terms**

Zeno, Boris Johnson, France, The North Sea, Radon, soap, bread, GDP, logical positivism, ...
tortoise, toothless, Conservative, nobel gas, philosopher, chemical element, ...
loves, owns, reacts with, voted for, ...

# Argument 2 revisited again

**Argument 2**

Zeno is a tortoise.
All tortoises are toothless.
Therefore, Zeno is toothless.

# Argument 2 revisited again

**Argument 2** **Valid**

Zeno is a tortoise.
All tortoises are toothless.
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# Argument 2 revisited again

**Argument 2** **Valid**

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**Argument 3**

Boris Johnson is a Conservative.
No Conservatives are Liberal Democrats.
Therefore, Boris Johnson is a Liberal Democrat.

# Argument 2 revisited again

**Argument 2** **Valid**

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Therefore, Zeno is toothless.

**Argument 3** **Not valid**

Boris Johnson is a Conservative.
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# Argument 2 revisited again

**Argument 2** **Valid**

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**Argument 3** **Not valid**

Boris Johnson is a Conservative.
No Conservatives are Liberal Democrats.
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**Argument 4**

Radon is a noble gas.
All noble gases are chemical elements.
Therefore, air is a chemical element.

# Argument 2 revisited again

**Argument 2** **Valid**

Zeno is a tortoise.
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Therefore, Zeno is toothless.

**Argument 3** **Not valid**

Boris Johnson is a Conservative.
No Conservatives are Liberal Democrats.
Therefore, Boris Johnson is a Liberal Democrat.

**Argument 4** **Not valid**

Radon is a noble gas.
All noble gases are chemical elements.
Therefore, air is a chemical element.

# Course overview

1: Validity; Introduction to Sets and Relations

2: Syntax and Semantics of Propositional Logic

3: Formalization in Propositional Logic

4: The Syntax of Predicate Logic

5: The Semantics of Predicate Logic

6: Natural Deduction

7: Formalization in Predicate Logic

8: Identity and Definite Descriptions

# Sets 1/2

**Characterisation**

A **set** is a collection of zero or more objects.

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- The set of positive integers:
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# Sets 2/2

**Fact about sets**

Sets are identical if and only if they have the same elements.

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**Example**

The following sets are all identical:

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- {Ringo, Ringo, Ringo, Lennon, Harrison, McCartney}

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- {Ringo, Ringo, Ringo, Lennon, Harrison, McCartney}
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- $\{x : x \text{ sang lead vocals on an Abbey Road track}\}$

# Ordered pairs

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$\langle$London, Munich$\rangle \neq \langle$Munich, London$\rangle$

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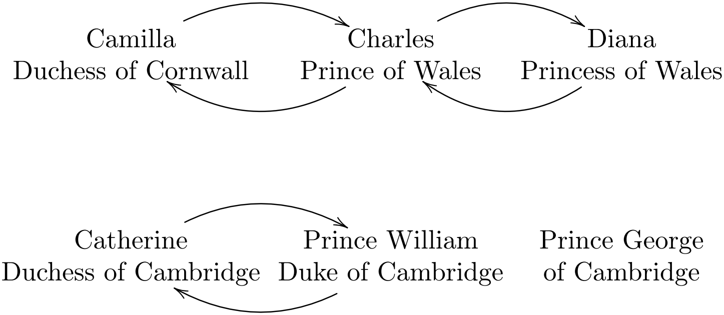
**Example**

$\langle \text{London, Munich} \rangle \neq \langle \text{Munich, London} \rangle$
$\{\text{London, Munich}\} = \{\text{Munich, London}\}$

# Relations


Camilla
Duchess of Cornwall
Charles
Prince of Wales
Diana
Princess of Wales
Catherine
Duchess of Cambridge
Prince William
Duke of Cambridge
Prince George
of Cambridge

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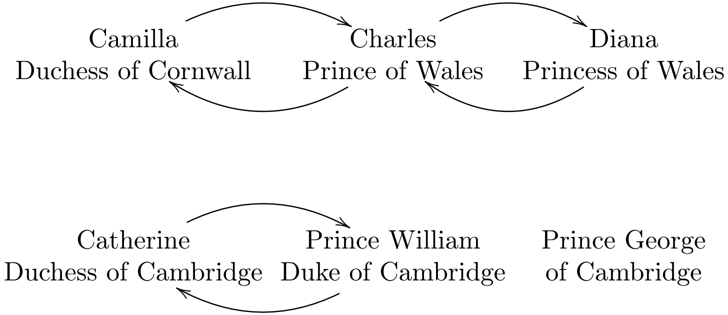


**The relation of *having married***

$\{ \quad , \quad , \quad , \quad , \quad , \quad , \quad \}$

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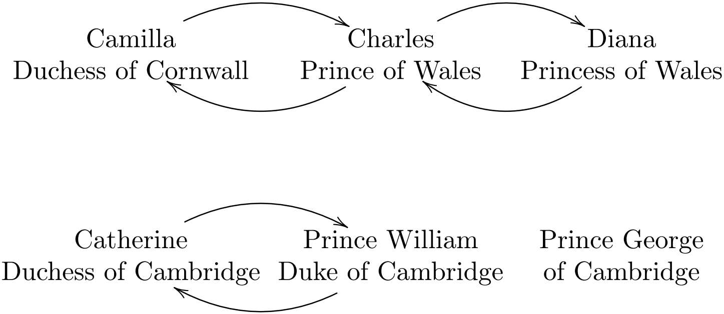


**The relation of *having married***

$\{\langle \text{Charles, Diana} \rangle,$ ,

, ,

, , $\}$

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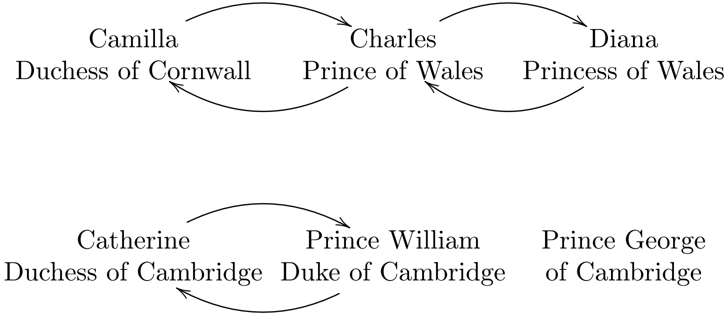


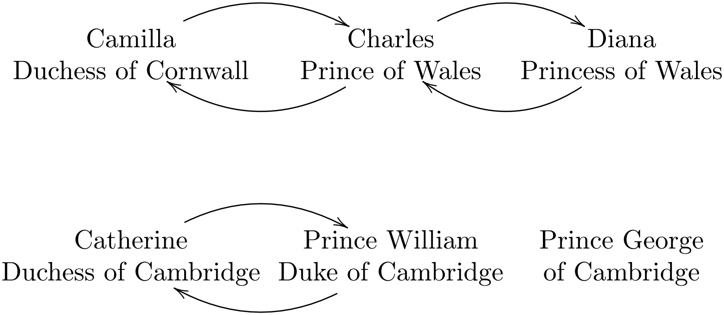

**The relation of *having married***

$$\{\langle \text{Charles, Diana} \rangle, \langle \text{Diana, Charles} \rangle,$$

$$,\qquad ,$$

$$,\qquad ,\quad \}$$

# Relations




**The relation of *having married***

$$\{\langle \text{Charles, Diana} \rangle, \langle \text{Diana, Charles} \rangle,$$
$$\langle \text{Charles, Camilla} \rangle, \qquad\qquad ,$$
$$\qquad , \qquad\qquad , \quad \}$$

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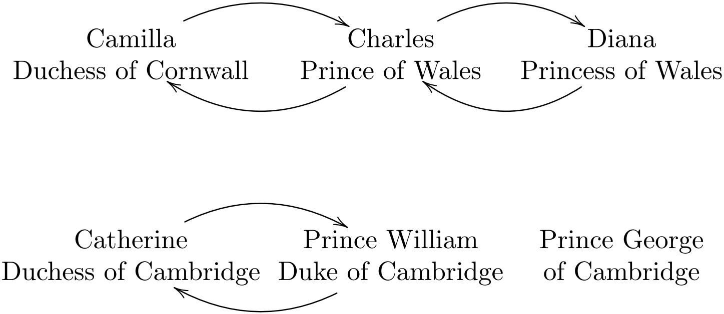


**The relation of *having married***

$$\{\langle \text{Charles, Diana} \rangle, \langle \text{Diana, Charles} \rangle,$$
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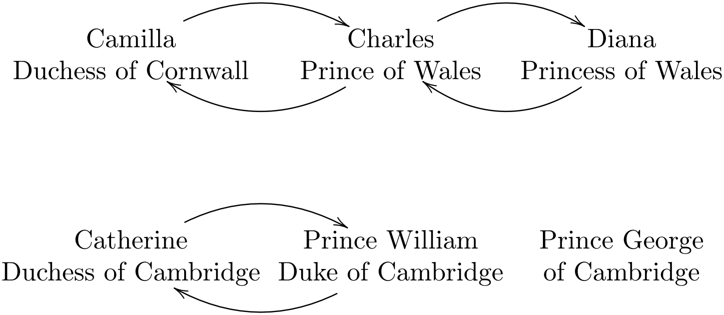
$$\qquad\qquad , \qquad\qquad , \quad \}$$

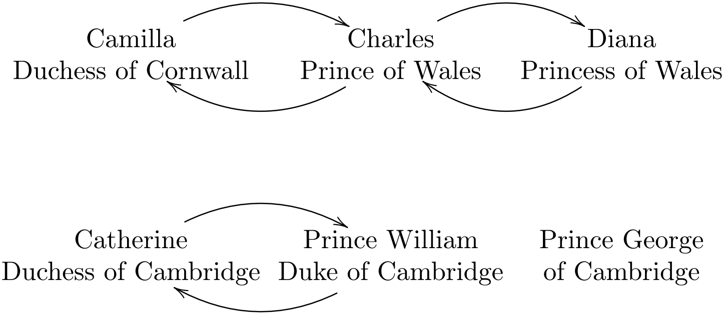
# Relations




## The relation of *having married*

$$
\begin{aligned}
\{&\langle \text{Charles, Diana}\rangle, \langle \text{Diana, Charles}\rangle,\\
&\langle \text{Charles, Camilla}\rangle, \langle \text{Camilla, Charles}\rangle,\\
&\langle \text{Kate, William}\rangle, \qquad\qquad\qquad , \quad \}
\end{aligned}
$$

# Relations




**The relation of *having married***

$$\{\langle \text{Charles, Diana}\rangle, \langle \text{Diana, Charles}\rangle,$$
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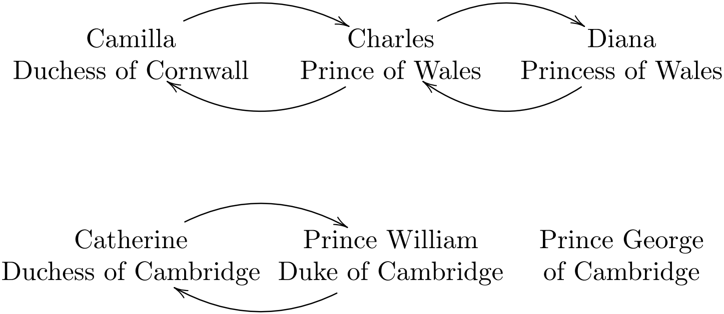



**The relation of *having married***

$$\{\langle \text{Charles, Diana} \rangle, \langle \text{Diana, Charles} \rangle, \\ \langle \text{Charles, Camilla} \rangle, \langle \text{Camilla, Charles} \rangle, \\ \langle \text{Kate, William} \rangle, \langle \text{William, Kate} \rangle, \ldots\}$$

# Worked example

Write down the following relation as a set of ordered pairs.
Draw its arrow diagram.



**The relation of *being countries in GB sharing a border***

# Worked example

Write down the following relation as a set of ordered pairs.
Draw its arrow diagram. 40

**The relation of *being countries in GB sharing a border***

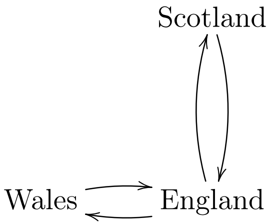
$$\{\langle \text{England, Scotland} \rangle, \langle \text{Scotland, England} \rangle,$$
$$\langle \text{England, Wales} \rangle, \langle \text{Wales, English} \rangle\}$$

# Worked example

Write down the following relation as a set of ordered pairs.
Draw its arrow diagram. 40

**The relation of *being countries in GB sharing a border***

$$\{\langle \text{England, Scotland}\rangle, \langle \text{Scotland, England}\rangle,$$
$$\langle \text{England, Wales}\rangle, \langle \text{Wales, English}\rangle\}$$



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A set $R$ is a **binary relation** if and only if it contains only ordered pairs.

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A set $R$ is a **binary relation** if and only if it contains only ordered pairs.

Informally: $\langle d, e \rangle \in R$ indicates that $d$ stands in $R$ to $e$.

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## Example

- The relation of *having married.*
  - $\{\langle \text{Kate, William}\rangle, \langle \text{Charles, Camilla}\rangle, \cdots\}$

# Relations

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- The empty set: $\emptyset$

# Properties of relations 1/3

## Definition (p. 9)

A binary relation $R$ is **reflexive on a set $S$** iff:

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**Example** **Reflexive on the set of human beings**

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- $\{\langle 1, 1 \rangle, \langle 2, 2 \rangle, \langle 1, 3 \rangle\}$

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**Example** — **Not reflexive on $\{1, 2, 3\}$**

- $\{\langle 1, 1 \rangle, \langle 2, 2 \rangle, \langle 1, 3 \rangle\}$ — Reflexive on $\{1, 2\}$

## Reflexivity on $S$

Every point in $S$ has a "loop".

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**Example** **Symmetric on the set of human beings**

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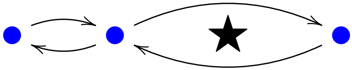

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- The relation of *being taller than*

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**Example** **Transitive on the set of human beings**

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**Example — Transitive on the set of human beings**

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**Example — Not transitive on this set**

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Every "double-step" between points in $S$ has a "one-step shortcut".

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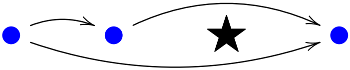

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# Functions

## Definition (p. 14)

A binary relation $F$ is a **function** iff for all $d, e, f$:

- if $\langle d, e \rangle \in F$ and $\langle d, f \rangle \in F$ then $e = f$.

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Informally, everything stands in $F$ to at most one thing.

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Informally, everything stands in $F$ to at most one thing.

## Example

- The function that squares positive integers.
  $\{\langle 1, 1\rangle, \langle 2, 4\rangle, \langle 3, 9\rangle, \ldots\}$
  $\{\langle x, y\rangle : y = x^2, \text{ for } x \text{ a positive integer}\}$

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Everything stands in $F$ to at most one thing (“many-one” or “one-one”)

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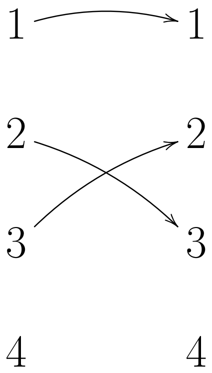

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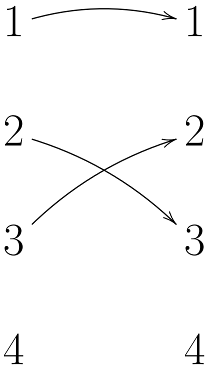


"one-one"
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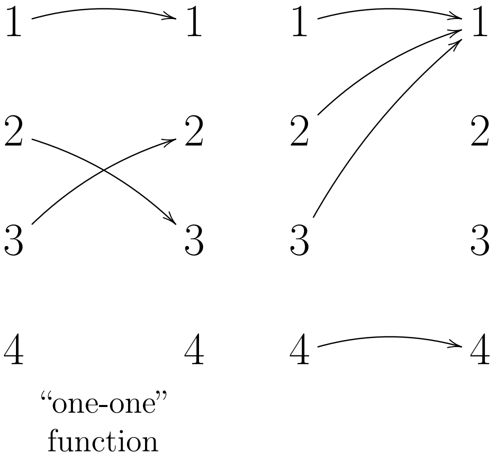

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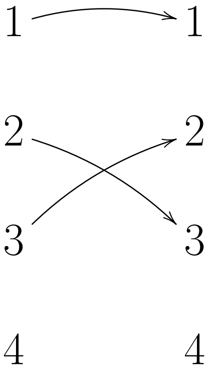
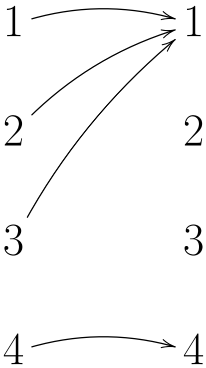



"one-one" function

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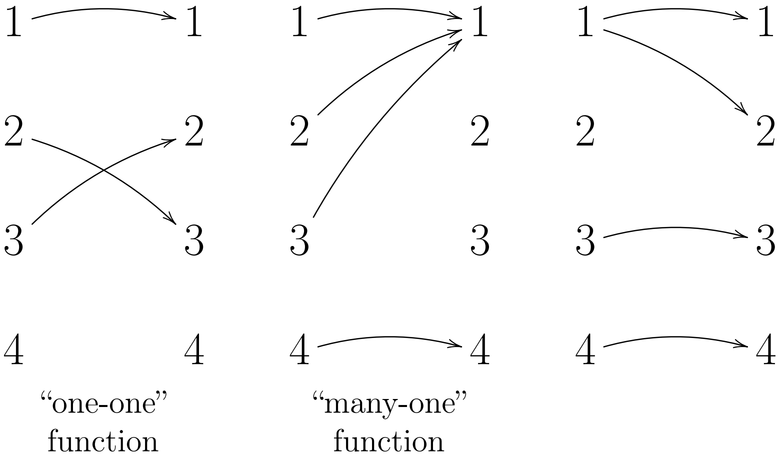

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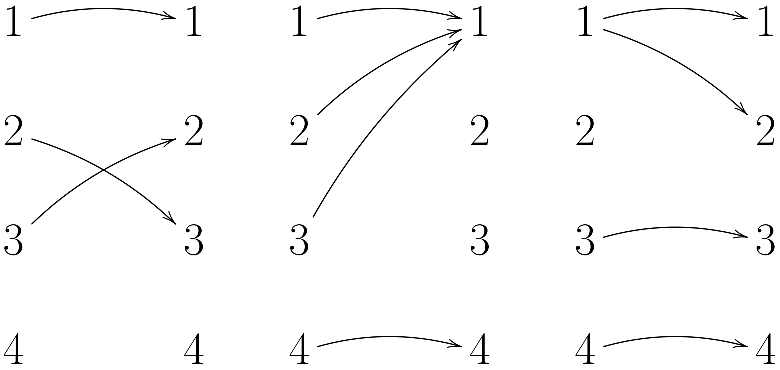


"one-one" function

"many-one" function

"one-many" not a function

# A “straightforward and elementary” example

(a) What is a binary relation?

(b) Consider the relation $R$ of *sharing exactly one parent*:

$$R = \{\langle d, e\rangle : d \text{ and } e \text{ share exactly one of their parents}\}$$

Determine whether $R$ is:

**(i)** reflexive on the set of human beings

**(ii)** symmetric on the set of human beings

**(iii)** transitive on the set of human beings

Explain your answers.

# A straightforward and elementary example

(a) What is a binary relation?

A binary relation is a set of zero or more ordered pairs.

# A straightforward and elementary example

(b) $R = \{\langle d, e\rangle : d$ and $e$ share exactly one of their parents$\}$

**(i)** Is $R$ reflexive on the set of human beings?

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I share two parents with myself, not one.

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(b) $R = \{\langle d, e\rangle : d$ and $e$ share exactly one of their parents$\}$

**(i)** Is $R$ reflexive on the set of human beings? No.
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(b) $R = \{\langle d, e\rangle : d \text{ and } e \text{ share exactly one of their parents}\}$

**(i)** Is $R$ reflexive on the set of human beings? No.
I share two parents with myself, not one.

**(ii)** Is $R$ symmetric on the set of human beings? Yes.
If human beings $d$ and $e$ share exactly one parent, clearly $e$ and $d$—the very same people—share exactly one parent too.

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If human beings $d$ and $e$ share exactly one parent, clearly $e$ and $d$—the very same people—share exactly one parent too.

**(iii)** Is $R$ transitive on the set of human beings? No.
For example, my maternal half-sister Rachel and I share exactly one parent, and me and my paternal half-sister Debby share exactly one parent, but Rachel and Debby share no parents.

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