Hybrid-relativism and revenge

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Outline

I. Absolute generality – an introduction

II. Relativist-friendly modality

III. Hybrid relativism – regimenting the argument from IE

IV. A revenge problem for hybrid relativism

Absolute generality

Quantifiers are often restricted:

(1) No donkey talks.

(2) 40% off **EVERYTHING!**

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- ∀-absolutist: some domain comprises absolutely everything
- ∀-relativist: no domain comprises absolutely everything

Relativism – the case from indefinite extensibility (IE)

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Given a domain M, let $r = \{x \in M : x \notin x\}$. Then $r \notin M$

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- suppose M is the intended domain of impure set theory ZFCU
- not every plurality in M forms a set (*M* not a set)

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Orthodox relativist response – no absolutely comprehensive M

- sequence of intended models of ZFCU with ever-larger domains: $M_0, M_1, M_2 \ldots$
- any plurality in M_α 'eventually' forms a set

How to regiment $\forall$-relativism?

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- hopeless: standard plural logic – e.g. PFO – refutes ¬CD
- CD is a consequence of an axiom of PFO – namely, PC:

Plural Comprehension: any $\phi(x)$ defines a ‘plurality’

PC: $\exists xx (xx \equiv_x \phi(x))$ ($xx \equiv_x \phi(x) := \forall x (x \prec xx \leftrightarrow \phi(x))$)

- response: frame relativism in a modal language

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Relativist-friendly modality

The plural language of ZFCU: $\mathcal{L}$

- singular and plural quantifiers $\forall v$, $\forall vv$
- non-logical predicates: β and $\in$

MT-structure: $\mathcal{M} := \langle M, S, E \rangle$ $(M \neq \emptyset, S \subseteq M, E \subseteq M \times M)$

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A modal extension: $\mathcal{L}^{\Diamond}$

- adds 'forward-' and 'backwards-looking' modal operators

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MT-hierarchy: $\mathcal{H} := \{\mathcal{M}_\alpha\}_{\alpha<\lambda}$ $\qquad$ (λ is a limit ordinal)

- $\mathcal{M}_\alpha$ is an MT-structure or $\langle \emptyset, \emptyset, \emptyset \rangle$ $\qquad$ ($M_\alpha \neq \emptyset$ for some α)
- $\alpha < \beta \Rightarrow \mathcal{M}_\alpha$ is a substructure of $\mathcal{M}_\beta$

(i.e. $M_\alpha \subseteq M_\beta$, $E_\beta \cap M_\alpha = E_\alpha$, etc.)

Kripke semantics

Given $\mathcal{H} := \{\mathcal{M}_\alpha\}_{\alpha<\lambda}$, $\alpha < \lambda$, suitable σ:

- $u \in v$ is true$_{\alpha,\sigma}$ iff $\langle \sigma(u), \sigma(v) \rangle \in E_\alpha$ (similarly for β, $\prec$, $=$)
- $\forall v \psi$ is true$_{\alpha,\sigma}$ iff $\forall a \in M_\alpha$: ψ is true$_{\alpha,\sigma[v/a]}$
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  - $\Box$ not semantically reducible to $\forall \beta$.
  - 'non-circumstantial' modality (despite 'process'-metaphor)

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- disclaimers:
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  - ‘non-circumstantial’ modality (despite ‘process’-metaphor)
- target: ‘hybrid’ relativists, who accept $\Box$ (or $\Box_>$) as primitive

A logic for $\mathcal{L}^{\Diamond}$

Bimodal logic – MPFO

- negative, free PFO (with PC) + basic tense logic (defined $E!v$ and $E!vv$)

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- H: $M\psi_1 \wedge M\psi_2 \rightarrow (M(\psi_1 \wedge \psi_2) \vee M(\psi_1 \wedge M\psi_2) \vee M(\psi_2 \wedge M\psi_1))$ $(M = \Diamond_>, \Diamond_<)$
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- CBF: $\Box_{>}\forall \boldsymbol{v}\psi \rightarrow \forall \boldsymbol{v}\Box_{>}\psi$ ($\boldsymbol{v}$ singular or plural)
- STA-$\in$: $\Box\forall x\forall y(\Diamond x \in y \rightarrow x \in y))$ (also for β, $=$, $\prec$)

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Derived unimodal axioms

- $\Box$ conforms to S5

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- hybrid relativism = ∀-relativism + □∀-absolutism (‘HRism’)

- ∀-relativism: ∀ never has an absolutely comprehensive domain
- □∀-absolutism: □∀ ‘generalizes’ about absolutely everything

How to regiment ∀-relativism? – revisited

Recall the second attempt:

Relativism (try 2): no plurality-domain comprises everything

$\neg$CD : $\neg\exists xx \forall x(x \prec xx)$

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HRists – who typically accept PC – may give a modal alternative:

Relativism (final version):
absolutely no plurality-domain comprises absolutely everything

$\neg\text{CD}^{\Diamond}$: $\neg \Diamond \exists xx \Box \forall x (x \prec^{\Diamond} xx)$

- MPFO neither proves nor refutes $\neg\text{CD}^{\Diamond}$

The argument from IE – revisited

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Consider e.g. the plurality *oo* of ordinals (in the domain M)

Answer 1 – the paradoxes: e.g. $\textsc{zfcu} \vdash \neg\exists s \forall x(x \in s \leftrightarrow \textsc{ord}(x))$

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Relativist objects – ZFCU tells us about collectedness not collectability

- *oo* is not collect<u>ed</u> $(\text{ZFCU} \vdash_{\text{PFO}} \neg\exists s(s \equiv oo))$
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Relativist objects – why <u>that</u> order-type?

- it's coherent for *oo* to form a set (at a stage outside $\mathcal{S}$)
- then there'd be a longer sequence $\mathcal{S}^+$
- why privilege $\mathcal{S}$ over $\mathcal{S}^+$?

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- answer: precisely when $\phi^{\Diamond}(x)$ is not indefinitely extensible

- EXT$_x[\phi^{\Diamond}(x)] := \Diamond_{>} \exists x(\phi^{\Diamond}(x) \wedge \Box_{<} \neg E!x)$ $(E!x := x = x)$

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- answer: precisely when $\phi^{\Diamond}(x)$ is not indefinitely extensible

- $\textsc{ext}_x[\phi^{\Diamond}(x)] := \Diamond_> \exists x(\phi^{\Diamond}(x) \wedge \Box_< \neg E!x)$ $(E!x := x = x)$

Inextensibility: $\vdash_{\text{MPFO}} \neg \Box\, \textsc{ext}_x[\phi^{\Diamond}(x)] \leftrightarrow [\exists xx(xx \equiv_x \phi(x))]^{\Diamond}$

more generally, holds for 'invariant' $\psi(x)$. $(\textsc{inv}[\psi] := \Box \forall \vec{v}(\Box \psi \vee \Box \neg\psi))$

Outline

I. Absolute generality – an introduction

II. Relativist-friendly modality

III. Hybrid relativism – regimenting the argument from IE

IV. A revenge problem for hybrid relativism

Hybrid-relativism and revenge

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- **thorough-going relativism:** = $\forall$-relativism + $\Box\forall$-relativism ("TGRism")

TGRist – revenge problem for HR

- introduce $\boxdot$, intended to be broader than $\Box$
- frame a new version of argument from IE
- argue that non-comprehensibility isn't much easier to explain than non-collectability

More relativist-friendly modality

An extended modal language – $\mathcal{L}^{\diamond}$

- add $\boxdot_{>}$ and $\boxdot_{<}$ to $\mathcal{L}^{\diamond}$ ($\Box$ now sole primitive)
- define $\boxdot\psi := \boxdot_{>}\psi \land \psi \land \boxdot_{<}\psi$

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Extended MT-hierarchy: $\mathcal{H}_{\lambda,\lambda^+} := \langle \mathcal{H}, \lambda \rangle$

- $\mathcal{H}_\lambda := \{\mathcal{M}_\alpha\}_{\alpha<\lambda}$ is an MT-hierarchy
- $\mathcal{H}_{\lambda^+} := \mathcal{H} = \{\mathcal{M}_\alpha\}_{\alpha<\lambda^+}$ is an MT-hierarchy
- $\lambda \leq \lambda +$

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Given $\mathcal{H}_{\lambda,\lambda^+}$, $\alpha < \lambda^+$, suitable σ:

- $\boxdot\psi$ is true$_{\alpha,\sigma}$ iff $\forall\beta$: ψ is true$_{\beta,\sigma}$ $(\beta < \lambda^+)$
- similarly for $\boxdot_{>}$ and $\boxdot_{<}$
- $\Box\,\psi$ is true$_{\alpha,\sigma}$ iff $\forall\beta < \lambda$: ψ is true$_{\beta,\sigma}$

A logic for $\mathcal{L}^{\diamond}$

Trimodal logic – MPFO^{+}

- MPFO for $\boxdot_{>}$ and $\boxdot_{<}$.

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- D: $\Box\psi \to \Diamond\psi$ ($\Box\psi \to \psi$ may fail above λ)
- 4$_b$: $\Box\psi \to \boxdot\Box\psi$
- 5$_b$: $\Diamond\psi \to \boxdot\Diamond\psi$

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A logic for $\mathcal{L}^{\odot}$

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- E$_1^-$: $\Box\forall v\Diamond E!v$ ($\Diamond E!v$ may fail)

A revenge argument

- TGRist conclusion:

New Items: not $\boxdot$-absolutely every item is a $\Diamond$-potential item

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Liberal Comprehensibility: each +-invariant condition $\psi(x)$ that is not +-extensible is +-comprehensible.

LC: $+\text{INV}[\psi(x)] \wedge \neg{+\text{EXT}_x}[\psi(x)] \rightarrow \diamond \exists xx(xx \equiv^{\diamond}_x \psi(x))$

$+\text{INV}[\psi] := \boxdot \forall \vec{v} (\boxdot \psi \vee \boxdot \neg \psi)$ $+\text{EXT}_x[\psi(x)] := \diamond \exists x (\psi(x) \wedge \Box \neg E!x)$

- HRist response: deny LC

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What makes non-comprehensible $\phi^{\diamond}(x)$ non-comprehensible?

Answer (Linnebo): some conditions have 'an inexhaustible character that renders them intrinsically unsuitable for defining sets.'

('Potential Hierarchy of Sets', p. 213)

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- 'inexhaustible character' – or, at least $\Box\, \textsc{ext}_x[\textsc{ord}^{\Diamond}(x)]$
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- TGRist: yes, since $\neg{+}\mathrm{EXT}_x[\textsc{ord}^{\Diamond}(x)]$ (so $\neg \boxdot\, \mathrm{EXT}_x[\textsc{ord}^{\Diamond}(x)]$)

How convincing is this HRist explanation?

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- relativist objects – why that hierarchy?
  - it's coherent for $\textsc{ord}^{\Diamond}(x)$ to be +-comprehended (outside $\mathcal{H}$)
  - then there'd be a more inclusive hierarchy $\mathcal{H}^+$
  - why privilege $\mathcal{H}$ over $\mathcal{H}^+$?