LP

→	1	0	#
1	1	0	#
0	1	1	1
#	1	#	#

LP*

→	1	0	#
1	1	0	0
0	1	1	1
#	1	0	#

$V^*(\varphi \to \psi) = 0$ iff $V^*(\varphi) > V^*(\psi)$
$\Rightarrow V^*(\varphi) = 1$ & $V^*(\psi) = 0$

2.

a.i Claim: $P \to Q,\ R \to S \models (P \to S) \vee (R \to Q)$

$d, d' = 1$ or #

1 d ? d w/ 1 0 0 0 1 0 0

Pf Suppose $V(P \to Q), V(R \to S) \neq 0$ [V^*, V_2 for same int I]

and $V(P \to S \vee R \to Q) = 0$

$V(P \to S) = V(R \to Q) = 0$

$V(P) = 1$ and $V(Q) = 0$

$\therefore V(P \to Q) = 0$

LP

$\rightarrow$	1	0	#
1	1	0	#
0	1	1	1
#	1	#	#

LP*

$\rightarrow$	1	0	#
1	1	0	0
0	1	1	1
#	1	0	#

$V^*(\varphi \rightarrow \psi) = 0$ iff $V^*(\varphi) > V^*(\psi)$ $\Rightarrow V^*(\varphi) = 1$ or $V^*(\psi) = 0$.

2

a. i <u>Claim:</u>

$d, d' = 1$ or #

$P \rightarrow Q,\ R \rightarrow S \models (P \rightarrow S) \vee (R \rightarrow Q)$

P	→	Q	, R	→	S		(P	→	S)	∨	(R	→	Q)	
1	d	1			d'	LP*	1	0	0	0		0	?	] $V(P) = 1$
1	d	1					1	0	#	0		0	?	
			0	d'	0		#	0	0	0	?	0		] $V(S) = 0$

Pf Suppose for ∴ $V^*(P \rightarrow Q), V^*(R \rightarrow S) \neq 0$

but $V^*((P \rightarrow S) \vee (R \rightarrow Q)) = 0$

∴ $V^*(P \rightarrow S) = V^*(R \rightarrow Q) = 0$

Since $V^*(P \rightarrow S) = 0$

∴ $V^*(P) = 1$ or $V^*(S) = 0$

Goto ⊕

⊕ Case 1: $V^*(P) = 1$.

Since $V^*(P \rightarrow Q) \neq 0$, $V^*(Q) = 1$

∴ $V^*(R \rightarrow Q) = 0$

Case 2: $V^*(S) = 0$

Since $V^*(R \rightarrow S) \neq 0$, $V^*(R) = 0$

∴ $V^*(R \rightarrow Q) = 0$

2

LP

$\to$	1	0	#
1	1	0	#
0	1	1	1
#	1	#	#

LP*

$\to$	1	0	#
1	1	0	0
0	1	1	1
#	1	0	#

$V^*(\varphi \to \psi) = 0$ iff $V^*(\varphi) > V^*(\psi)$

$\Rightarrow V^*(\varphi) = 1$ or $V^*(\psi) = 0$.

a.ii <u>Claim:</u> $P \to Q,\ P \nvDash_{LP} Q$

0   #   0

Countermodel.

Set $I(P) = \#$, $I(Q) = 0$.

Claim: $P \to Q,\ P,\ \vDash_{LP^*} Q$

$d, d' \neq 0$ d d 0 d' 0

Suppose $V^*(P \to Q)$, $V^*(P) \neq 0$

and $V^*(Q) = 0$.

$\therefore V^*(P) = 0$ ※

LP

$\rightarrow$	1	0	#
1	1	0	#
0	1	1	1
#	1	#	#

LP*

$\rightarrow$	1	0	#
1	1	0	0
0	1	1	1
#	1	0	#

$V^*(\varphi \rightarrow \psi) = 0$ iff $V^*(\varphi) > V^*(\psi)$
$\Rightarrow V^*(\varphi) = 1$ or $V^*(\psi) = 0$.

2

a. iii

Claim: $P \vee Q,\ \sim P \models Q$

0   # #   0

Countermodel (for LP & LP*)

$I(P) = \#,\ I(Q) = 0.$

2

LP

$\rightarrow$	1	0	#
1	1	0	#
0	1	1	1
#	1	#	#

LP*

$\rightarrow$	1	0	#
1	1	0	0
0	1	1	1
#	1	0	#

$V^*(\varphi \to \psi) = 0$ iff $V^*(\varphi) > V^*(\psi)$ $\Rightarrow V^*(\varphi) = 1$ or $V^*(\psi) = 0$.

b.i. Claim: if $\models_{LP^*} \varphi$, then $\models_{PL} \varphi$

STP: if $\not\models_{PL} \varphi$, then $\not\models_{LP^*} \varphi$

Suppose $\not\models_{PL} \varphi$. $\therefore \exists$ biv. I s.t. $V_I(\varphi) = 0$

From Lemma $V^*_I(\varphi) = V_I(\varphi) = 0$

$\therefore \not\models_{LP^*} \varphi$.

Lemma for biv. I $V^*_I(\varphi) = V_I(\varphi)$.

pf by induction.

Base case: $\varphi = \alpha$, a S.L.

$V^*_I(\alpha) = I(\alpha) = V_I(\alpha)$

IH. Suppose $V^*_I(\psi) = V_I(\psi)$ for ψ less complex than φ.

$\varphi = \psi_1 \to \psi_2$. By IH, $V_I(\psi_i) = V^*_I(\psi_i) = 0$ or 1.

By inspecting truth tables, $V_I(\psi_1 \to \psi_2) = V^*_I(\psi_1 \to \psi_2)$.

Sim for $\sim, \wedge, \vee$.

2. a. i. $r(\varphi) := \Diamond(\varphi \wedge \Box\varphi) \wedge \Diamond(\sim\varphi \wedge \Box\sim\varphi)$

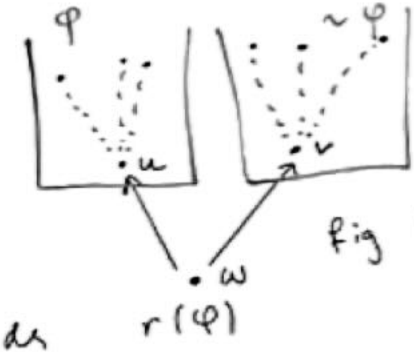


fig 1

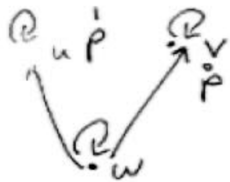
ii. From fig. 1, there is a 'irreversible choice' at w, one can take the 'φ-track' at u and then φ always holds at u-accessible worlds or the '$\sim\varphi$-track' and $\sim\varphi$ always holds at v-accessible worlds.

iii. KS became PM in 2024

Train T is on the left track.

2. a. $r(p) := \Diamond(p \wedge \Box p) \wedge \Diamond(\sim p \wedge \Box \sim p)$

b.



p is a RS at w in the following S4-model

$W = \{w, u, v\}$

$R = \{\langle w,u \rangle, \langle w,v \rangle, \ldots\}$

$I(p, u) = 1$

and $I(\alpha, x) = 0$ otherwise.

fig 1

2. a. i. $r(p) := \Diamond(p \wedge \Box p) \wedge \Diamond(\sim p \wedge \Box \sim p)$

Answer: No.

b. ii Suppose $r(\varphi)$ holds at w

in a model in which $\Diamond\Box\psi \to \Box\Diamond\psi$ is valid (for X)

$\Box \sim r(\varphi)$

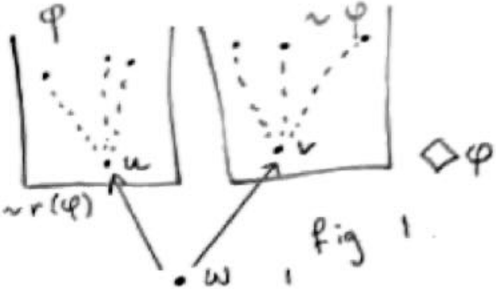


fig 1

$r(\varphi)$ $\Diamond\Box\varphi$

$\Box\Diamond\varphi$

$\therefore \exists u, v$ s.t. wRu and wRv that meet (i) and (ii)

By (i) $V(\Box\varphi, u) = 1$ $\therefore V(\Diamond\Box\varphi, w) = 1$

$\therefore V(\Box\Diamond\varphi, w) = 1$ since $\Diamond\Box\psi \to \Box\Diamond\psi$ is valid.

$\therefore V(\Diamond\varphi, v) = 1$ since wRv

$\therefore V(\varphi, v') = 1$ for some v' s.t. vRv'

$\therefore$ ⨳ (ii).

b. iii Contingent. If φ is RS at w

then φ is not a RS at u

and also φ is not a RS at u' s.t. uRu'.

$\therefore$ Not necessarily contingent.

Not necessarily non-contingent.

DT: if $\Gamma, \varphi \vdash \psi$, then $\Gamma \vdash \varphi \to \psi$

Contr 1: if $\vdash \sim\varphi \to \sim\psi$, then $\vdash \psi \to \varphi$

Contr 2*: if $\vdash \varphi \to \psi$, then $\vdash \sim\psi \to \sim\varphi$

EFQ: $\varphi, \sim\varphi \vdash \psi$

Neg imp*: $\sim(\varphi \to \psi) \vdash \sim\psi$

* Req. pr.
[$\vdash \varphi \to \sim\sim\varphi$
$\vdash \sim\sim\varphi \to \varphi$]

: $\vdash (P \to Q) \to ((Q \to R) \to (P \to R))$

By DT×2, STP: $P \to Q, Q \to R \vdash P \to R$

By DT again (†) $P, P \to Q, Q \to R \vdash R$.

Axiomatic proof of (†)

P	ass
$P \to Q$	ass
Q	MP
$Q \to R$	ass
R	MP

DT. if $\Gamma, \varphi \vdash \psi$, then $\Gamma \vdash \varphi \to \psi$

Contr 1. if $\vdash \sim\varphi \to \sim\psi$, then $\vdash \psi \to \varphi$

Contr 2.* if $\vdash \varphi \to \psi$, then $\vdash \sim\psi \to \sim\varphi$

EFQ $\quad \psi, \sim\psi \vdash \varphi$

Neg and* $\sim(\varphi \to \psi) \vdash \sim\psi$

* Rely on. [$\vdash \varphi \to \sim\sim\varphi$
$\vdash \sim\sim\varphi \to \varphi$]

ie $\varphi \to \sim\sim\varphi$

<u>Lemma</u> $\vdash \sim\sim\varphi \to \varphi$ Pf of Lemma, by DT: STP $\sim\sim\varphi \vdash \varphi$

By PL3 $\quad \sim\sim\varphi \vdash (\sim\varphi \to \sim\underbrace{\psi}_{\sim\varphi}) \to ((\sim\varphi \to \underbrace{\psi}_{\sim\psi}) \to \varphi)$ (*)

Now note $\sim\varphi \vdash \sim\varphi \;\therefore\; \vdash \sim\varphi \to \sim\varphi$ (DT)

Also $\sim\sim\varphi, \sim\varphi \vdash \sim\sim\varphi \quad \sim\sim\varphi \vdash \sim\varphi \to \sim\sim\varphi$ (DT)

So from (*), CUT and MP x2 $\quad \sim\sim\varphi \vdash \varphi \;\therefore\; \vdash \sim\sim\varphi \to \varphi$ (DT)

□ Lemma.

DT. if $\Gamma, \varphi \vdash \psi$, then $\Gamma \vdash \varphi \to \psi$

Contr 1. if $\vdash \sim\varphi \to \sim\psi$, then $\vdash \psi \to \varphi$

Contr 2*. if $\vdash \varphi \to \psi$, then $\vdash \sim\psi \to \sim\varphi$

EFQ. $\varphi, \sim\varphi \vdash \psi$

Neg cond.* $\sim(\varphi \to \psi) \vdash \sim\psi$ [ALT. $\sim(\varphi \to \psi) \vdash \varphi$]

* Reg m.
[$\vdash \varphi \to \sim\sim\varphi$
$\vdash \sim\sim\varphi \to \varphi$]
↑
we have now proved these two.

ii. $P \to \sim\sim P$

<u>Lemma</u> $\vdash \sim\sim\varphi \to \varphi$ □.

From Lemma, $\vdash \sim\sim\sim P \to \sim P$ ∴ $\vdash P \to \sim\sim P$.
Contr 1

iii. $\sim(P \to Q) \to (Q \to P)$.

From Neg cond $\sim(P \to Q) \vdash \sim Q$ (†)

<u>Claim</u> $\sim Q \vdash Q \to P$

Pf By EFQ $\sim Q, Q \vdash P$

∴ $\sim Q \vdash Q \to P$ (‡)
DT

□ Claim

From (†) & (‡) $\sim(P \to Q) \vdash Q \to P$

∴ $\vdash \sim(P \to Q) \to (Q \to P)$.
DT

b: $\vdash \exists x(Fx \wedge Gx) \rightarrow (\exists x Fx \wedge \exists x Gx)$ A → B, A → C / A → (B ∧ C)

By PL, STP $\vdash \exists x(Fx \wedge Gx) \rightarrow \exists x Fx$ (A)

and $\vdash \exists x(Fx \wedge Gx) \rightarrow \exists x Gx$ (B)

Proof of (A) RTP: $\vdash \sim\forall x \sim(Fx \wedge Gx) \rightarrow \sim\forall x \sim Fx$

By PL, STP: $\vdash \forall x \sim Fx \rightarrow \forall x \sim(Fx \wedge Gx)$ (**)

A pf of (**)

$\vdash \forall x \sim Fx \rightarrow \sim Fx$

$\vdash \sim Fx \rightarrow \sim(Fx \wedge Gx)$ PL

$\vdash \forall x \sim Fx \rightarrow \sim(Fx \wedge Gx)$ PL

$\vdash \forall x \sim Fx \rightarrow \forall x \sim(Fx \wedge Gx)$ UG, PCL, MP

Pf of (B) Similarly.

b : $\forall x (Fx \leftrightarrow x = a) \rightarrow \forall y \forall z (Fy \wedge Fz \rightarrow y = z)$

Note first $\vdash \forall x (Fx \leftrightarrow x = a) \rightarrow (Fy \leftrightarrow y = a)$

$\vdash \forall x (Fx \leftrightarrow x = a) \rightarrow (Fz \leftrightarrow z = a)$

Lemma 1 $y = a, \ a = z \vdash y = z$ $\quad \alpha = a$, $\beta = z$

Pf of Lemma 1 Apply I2 $\alpha = \beta \rightarrow (\varphi(\alpha) \rightarrow \varphi(\beta))$

$a = z \rightarrow (y = a \rightarrow y = z)$ Result shown by MP x2

Lemma 2: $z = a \vdash a = z$

Apply I2 $\alpha = \beta \rightarrow (\varphi(\alpha) \rightarrow \varphi(\beta))$ $\quad \alpha = z$, $\beta = a$

$\vdash z = a \rightarrow (z = z \rightarrow a = z)$ Result shown by MP x2.

$\vdash z = z$ RX.

From Lemma 1, 2, and PL $\vdash \forall x (Fx \leftrightarrow x = a) \rightarrow (Fy \wedge Fz \rightarrow y = z)$

By UG, PC2, MP $\vdash \forall x (Fx \leftrightarrow x = a) \rightarrow \forall z (Fy \wedge Fz \rightarrow y = z)$

By UG, PC2, MP $\vdash \forall x (Fx \leftrightarrow x = a) \rightarrow \forall y \forall z (Fy \wedge Fz \rightarrow y = z)$

VDQML: $\langle W, R, D, Q, I \rangle$ $D_w := Q(w) \subseteq D$, $D \neq \emptyset$.

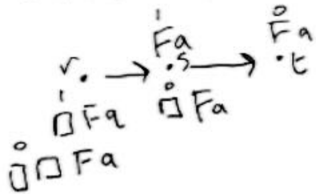
SQML: $\langle W, D, I \rangle$

a. i. The claim is false.

Let $\varphi = \Box Fa \to \Box\Box Fa$.

$\vDash_{SQML} \varphi$, since SQML validates the S5 axioms.

but $\nvDash_{VDQML} \varphi$.



Official countermodel

$W = \{r, s, t\}$

$R = \{\langle r, s \rangle, \langle s, t \rangle\}$

$D = \{u\}$ $Q(w) = D$ for each $w \in W$

$I(a) = u$ (sim. for any other constant)

$I(F) = \{\langle u, s \rangle\}$

VDQML: $\langle W, R, D, Q, I \rangle$ $D_w := Q(w) \subseteq D$, $D \neq \emptyset$.

SQML: $\langle W, D, I \rangle$

a ii. The claim is true.

Given SQML model $M = \langle W, D, I \rangle$

Consider VDQML model $M^P = \langle W, R, D, Q, I \rangle$ s.t. $R = \{\langle u, v \rangle : u, v \in W\}$

$Q(w) = D$ for each $w \in W$.

Lemma: $V_M(\varphi, w) = V_{M^P}(\varphi^P, w)$

Suppose $\nvDash_{SQML} \varphi$. Then φ is false at a world w in some SQML M $\therefore$ φ^P is false at w in M^P (by Lemma)

$\therefore \nvDash_{VDQML} \varphi^P$. So the claim follows by contraposition.

VDQML: $\langle W, R, P, Q, I \rangle$ $D_w := Q(w) \subseteq D$, $D \neq \emptyset$.

SQML: $\langle W, D, I \rangle$

a ii. The claim is true.

Given SQML model $M = \langle W, D, I \rangle$ (This can be any SQML-model)

Consider VDQML model $M^P = \langle W, R, P, Q, I \rangle$ (determined by choice of M).

s.t. $R = \{\langle u, v \rangle : u, v \in W\}$

$Q(w) = D$ for each $w \in W$.

Lemma: (*) $V_M(\varphi, w) = V_{M^P}(\varphi^P, w)$

Pf by induction on φ.

Base case: $\varphi = \Pi\alpha_1, \dots, \alpha_n$. $\varphi^P = \Pi\alpha_1, \dots, \alpha_n$.

$\langle [\alpha_1]_g, \dots [\alpha_n]_g, w \rangle \in I(\Pi)$

$V_{M,g}(\varphi, w) = 1$ iff $\langle [\alpha_1]_g, \dots [\alpha_n]_g, w \rangle \in I(\Pi)$

iff $V_{M^P,g}(\varphi^P, w)$

- IH: Suppose (*) holds for formulas less complex than φ.
- $V_{M,g}(\forall\alpha\psi, w) = 1$

iff $\forall d \in D$ $V_{M, g^\alpha_d}(\psi, w) = 1$

iff $\forall d \in D$ $V_{M^P, g^\alpha_d}(\psi^P, w) = 1$

iff $V_{M^P, g}(\forall_P \alpha \psi^P, w) = 1$

- $\varphi = \sim\psi$. $V_{M,g}(\varphi, w) = 1$ iff $V_{M,g}(\psi, w) = 0$

(IH) iff $V_{M^P,g}(\psi^P, w) = 0$

iff $V_{M^P,g}(\varphi^P, w) = 1$.

- Similarly for $\varphi = \psi_1 \to \psi_2$.
- $\varphi = \Box\psi$ $V_{M,g}(\varphi, w) = 1$ iff $\forall w' \in W$ $V_{M,g}(\psi, w') = 1$

iff $\forall w'$ s.t. wRw' $V_{M,g}(\psi, w') = 1$.

IH iff $\forall w'$ s.t. wRw' $V_{M^P,g}(\psi^P, w') = 1$

iff $V_{M^P,g}(\varphi^P, w) = 1$.

w : $\overset{1}{\Box}\forall x\,(Fx \to Gx) \overset{0}{\longrightarrow} \overset{0}{\forall} x\,(\Diamond Fx \to \Diamond Gx)$

$\overset{1}{\Diamond} F\tfrac{x}{u} \overset{0}{\longrightarrow} \overset{0}{\Diamond} G\tfrac{x}{u}$

$D_w = \{u\}$

↓

v : $\overset{1}{\forall} x\,(Fx \to Gx)$ $\quad \overset{1}{F}\tfrac{x}{u} \quad \overset{0}{G}\tfrac{x}{u}$

$D_v = \emptyset$.

$W = \{w, v\}$, $R = \{\langle w, v\rangle\}$, $D = \{u\}$,
$I(F) = \{\langle u, v\rangle\}$, $I(G) = \emptyset$.

b. ii. The claim is true.

Suppose (for ※) that the formula is false at w in VDQML model M,

Then (writing V_g for $V_{M,g}$) $V_g(\Box\forall_p x(Fx \to Gx), w) = 1$ (†)

$V_g(\forall x(\Diamond Fx \to \Diamond Gx, w) = 0$

$\therefore \exists u \in D_w$ s.t. $V_{g^x_u}(\Diamond Fx \to \Diamond Gx, w) = 0$

$\therefore V_{g^x_u}(\Diamond Fx, w) = 1; V_{g^x_u}(\Diamond Gx, w) = 0.$

$\therefore \exists v$ s.t. Rwv $V_{g^x_u}(Fx, v) = 1 \therefore u \in I_v(F)$

But $V_{g^x_u}(Gx, v) = 0 \therefore u \notin I_v(G)$

From (†) $V_g(\forall_p x(Fx \to Gx), v) = 1$

$\therefore V_{g^x_u}(Fx \to Gx, v) = 1$

※ $V_{g^x_u}(Fx, v) = 1, V_{g^x_u}(Gx, v) = 0$

b.iii Countermodel.

$W = \{w\}$, $R = \{\langle w, w\rangle\}$, $D = \{u\}$, $D_w = \emptyset$

(any I will do).

5 $V_g(@\varphi, v, w) = V_g(\varphi, v, v)$ Note: Write V_g for $V^?_{M,g}$

↗ reference ↖ evaluation "@"

$V_g(X\varphi, v, w) = V_g(\varphi, w, w)$

$V_g(F\varphi, v, w) = 1$ iff $\forall v' \in W \cdot V_g(\varphi, v', w) = 1$

a.. Claim $V_g(\varphi, v, w) = V_g(\varphi, w, w)$ provided φ lacks @

Pf by induction on φ

Base case. $\varphi = \Pi \alpha_1 \ldots \alpha_n$

$V_g(\Pi \alpha_1 \ldots \alpha_n, v, w) = 1$ iff $\langle [\alpha_1]_g, \ldots, [\alpha_n]_g \rangle \in I_w(\Pi)$

iff $V_g(\Pi \alpha_1 \ldots \alpha_n, w, w) = 1$

$\varphi = \alpha = \beta$

$V_g(\alpha = \beta, v, w) = 1$ iff $[\alpha]_g = [\beta]_g$ iff $V_g(\alpha = \beta, w, w) = 1$.

5 Suppose for ψ less complex than φ, if ψ lacks @ then $V_g(\psi, v, w) = V_g(\psi, w, w)$ IH.

Suppose φ lacks @

Consider $\varphi = \forall x \psi$.

$V_g(\forall x \psi, v, w) = 1$ iff $\forall a \in D \; V_{g^x_a}(\psi, v, w) = 1$

(IH) iff $\forall a \in D \; V_{g^x_a}(\psi, w, w) = 1$

iff $V_g(\forall x \psi, w, w) = 1$

Similar arguments work for $\varphi = \neg\psi$ or $\varphi = \psi_1 \wedge \psi_2$, and $\varphi = \Box\psi$

• $\varphi = F\psi$

$V_g(F\psi, v, w) = 1$ iff $\forall v' \in W \; V_g(\psi, v', w) = 1$

iff $V_g(F\psi, w, w) = 1$

• $\varphi = X\psi$

$V_g(X\psi, v, w) = V_g(\psi, w, w) = V_g(X\psi, w, w)$

Let w be a world in an S5ML model.

$V_g(F@\varphi, w, w) = 1$

iff $\forall w' \in W \; V_g(@\varphi, w', w) = 1$

iff $\forall w' \in W \; V_g(\varphi, w', w') = 1$

iff $\forall w' \in W \; V_g(\varphi, w, w') = 1$ by a ;

iff $V_g(\Box\varphi, w, w) = 1$

$\therefore \models_{2D} F@\varphi \leftrightarrow \Box\varphi$

Can we get a countermodel?

$$\Diamond x \forall x (@Hx \to \Box Gx) \to \forall x (\Box Hx \to @Gx) \quad u \in D$$

$$\Box H\underline{x} \to @G\underline{x}$$

$$x \forall x (@Hx \to \Box Gx) \quad \text{Probably not}$$

$$(@H\underline{x} \to \Box G\underline{x})$$

Suppose for $\cdot X$ $V_g(\Diamond x \forall x(@Hx \to \Box Gx), w, w) = 1$ (*) for any $w' \in W$

and $V_g(\forall x(\Box Hx \to @Gx), w, w) = 0$

$\therefore \exists u \in D$ s.t. $V_{g_u^x}(\Box Hx, w, w) = 1$ $u \in I_w(H)$ &

$V_{g_u^x}(@Gx, w, w) = 0$ $u \notin I_w(G)$

From (*) $\exists w^*$ s.t.

$V_g(x \forall x(@Hx \to \Box Gx), w, w^*) = 1$

$\therefore V_g(\forall x(@Hx \to \Box Gx), w^*, w^*) = 1$

$V_{g_u^x}((@Hx \to \Box Gx), w^*, w^*) = 1$ (‡)

$u \in I_{w^*}(H)$ $V_{g_u^x}(@Hx, w^*, w^*) = 1$ from

$u \notin I_w(G)$ $\therefore V_{g_u^x}(\Box Gx, w^*, w^*) = 0$ X (‡) ?

$$\Box(\exists x\, @Hx \to \exists x \Diamond Gx) \to \Diamond(\exists x Hx \to \exists x Gx)$$

Claim: GJD valid.

Pf: Suppose (for X) the formula is false at v, w

$V_g(\Box(\exists x\, @Hx \to \exists x \Diamond Gx, v, w) = 1$

and $V_g(\Diamond(\exists x Hx \to \exists x Gx, v, w) = 0$ (*)

$V_g(\exists x\, @Hx \to \exists x \Diamond Gx, v, v) = 1$ ∴ there are 2 cases

Case 1: $V_g(\exists x\, @Hx, v, v) = 0$ ∴ there is no $d \in D$ s.t. $d \in I_v(H)$

∴ $V_g(\exists x Hx, v, v) = 0$

∴ $V_g(\exists x Hx \to \exists x Gx, v, v) = 1$ X from (*)

Case 2: $V_g(\exists x \Diamond Gx, v, v) = 1$

∴ $\exists d \in D$ s.t. $V_{g^x_d}(\Diamond Gx, v, v) = 1$

∴ $V_{g^x_d}(Gx, v, w') = 1$ for some $w' \in W$

∴ $V_g(\exists x Gx, v, w') = 1$

∴ $V_g(\exists x Hx \to \exists x Gx, v, w') = 1$

X from (*)