Contingentist sets as potentialist properties

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Challenging the Infinite
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I. Contingentism and sets of possibilia

II. Potentialism and properties

III. Psets

IV. Objections and replies

Contingentism

– is existence contingent or necessary?

Necessitism [NNE-ism]: existence is necessary

(NNE) $\Box\forall x\Box\exists y(y = x)$

Contingentism: existence is contingent

– **motivation:** apparently incompossible individuals

me v. Tom WWI v. GEP

– $\text{incmp}(x, y) := \Diamond\,\text{ind}(x) \wedge \Diamond\,\text{ind}(y) \wedge \neg\Diamond(E!x \wedge E!y)$

– incompossibles motivate **'strong contingentism'**:

[SC-ism]: there could always be another individual

(SC) $\Box\forall xx\Diamond\exists y(\text{ind}(y) \wedge y \not\prec xx)$

Sets of possibilia | semantics 1

sc-ism: – semantic reflection motivates sets of possibilia:

Q: why is $\Diamond\exists x \Diamond\exists y\, \mathrm{incmp}(x, y)$ true? (cf. Peacocke, Gupta)

– **A:** because there is a suitable assignment, e.g.

$$\sigma : x \mapsto \text{me} \quad y \mapsto \text{Tom}$$

Sets of possibilia | semantics 2

– further motivation:

Q: how can we 'make sense' of the Goodman-Fritz sentence?

(GF) Most possible individuals are never born

– **A:** apply GQ semantics: e.g.

$|D \cap N| > |D - N|$ $D :=$ set of possible individuals

$N :=$ set of never-borns

(D – set of ALL possible individuals)

Sets of possibilia | metaphysics

– semantic reflection: motivates σ, D
– **but:** assuming sc-ism, there are no such sets

– because: **ontological dependence**

[OD-set]: necessarily, a set exists only if its elements exist

– assuming sc-ism:

$$\text{D exists} \Rightarrow_{\text{OD-set}} \text{all possible individuals exist} \Rightarrow \bot$$
$$\sigma \text{ exists} \Rightarrow_{\text{ZFU}} \{\text{me, Tom}\} \text{ exists} \Rightarrow_{\text{OD-set}} \text{me and Tom exist} \Rightarrow \bot$$

– what to do?

- **non-standard semantics –** avoid σ, D, etc.
- **bad metaphysics –** reject **[OD-set]**
- retain **standard semantics** without **bad metaphysics**?

Strategy #1 | sets of proxy-possibilia

Ersatzism: (e.g. Plantinga, Jager, . . .)

- possible individual, $x \mapsto x^*$, actual proxy (e.g. *being* x)
- set of possibilia $\mapsto$ set of proxies – e.g.:

$$\sigma^* : x \mapsto \text{me}^*, y \mapsto \text{Tom}^*$$

$$D^* = \text{set of proxy possible individuals}$$

– bad metaphysics?

– non-standard semantics? – right TCs, wrong subject?

- sc-ist: ‘I exist contingently’
- Proxy semantics: *being-me* is contingently exemplified

Strategy #2 | proxy-sets of possibilia

- Gupta hints at a different approach (1978, 465):

> ...even if our present conception of sets is such that on it the set {Tom, You} does not exist there does not appear to be any conceptual difficulty in introducing another conception of sets according to which such sets *do* exist.

- **my aim:** provide such a conception of ‘set’

– reductive proposal:

– proxy-sets or psets: (cf. Bealer: 'L-determinate')

- pset – set-like or 'stable' property (silent 'p')
- pmember: $x \in p$ understood as $\Diamond(x \text{ has } p)$

– example: pset of possible individuals:

$D^* =$ *being an individual* me $\in D^*$ Tom $\in D^*$

– plan:

- what is the underlying conception of properties?
- what makes psets setlike?
- what about non-standardness/badness objections?

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Potentialism | properties

– informally, properties are introduced stagewise:

– at each stage:

Comprehension: any $\phi(x)$ defines a concept [type $e \to t$]

Plenitude: each concept is, at every later stage, nominalized as a property [type e]

Priority: each property nominalizes some concept available at an earlier stage

Individuals: individuals are available at every stage

Intensionality: necessarily coextensive properties (concepts) are identical

– aim for this section:

- motivate a modal property theory – MPT
- to start with: using an extensional metatheory

Properties | $\langle W, d_\alpha \rangle$-hierarchy

– iterative sets: stage – $V_\alpha(U)$

U := set of urelements $\qquad V_\alpha(U) := U \cup \bigcup_{\beta<\alpha} \boldsymbol{P} V_\beta(U)$

– iterative properties: stage – $\langle W, d_\alpha \rangle$

W := the set of worlds $\qquad d_\alpha : w \mapsto \overbrace{d_\alpha(w)}^{\text{dom. of } w}$

$\langle W, d \rangle$:= initial frame $\qquad d_\alpha(w) := d(w) \cup \bigcup_{\beta<\alpha} \boldsymbol{\pi} d_\beta$

$\boldsymbol{\pi} d := \{p : p \sqsubseteq d\} \qquad p \sqsubseteq d := p : w \mapsto p(w) \subseteq d(w)$

MPT | Kripke semantics

$S ::= \pi x \mid x\,\eta\,y \mid Xx \mid \boldsymbol{x} = \boldsymbol{y} \mid \neg S \mid S \to S \mid \forall \boldsymbol{x} S \mid \Box S \mid \mathbb{G} S \mid \mathbb{H} S$

– boldface: $\boldsymbol{x}$ is x or X

$$\boldsymbol{D} := \overbrace{\bigcup_{w,\alpha} d_\alpha(w)}^{\text{outer dom.}} \qquad P_\alpha := \overbrace{\bigcup_{\beta<\alpha} \boldsymbol{\pi} d_\beta}^{\text{properties nominalized at } \alpha}$$

– formulas evaluated at $\langle w, \alpha \rangle$, $w \in W$, $\alpha \in \mathrm{On}$

– $a, p, Q \in \boldsymbol{D}$:

- $w, \alpha \vDash \pi p$ iff $p \in P_\alpha$ — 'a is a property'
- $w, \alpha \vDash a\,\eta\,p$ iff $p \in P_\alpha$ and $a \in p(w)$ — 'a exemplifies p'
- $w, \alpha \vDash Qa$ iff $Q \in \boldsymbol{\pi} d_\alpha$ and $a \in Q(w)$

MPT | modal operators / modalization

- $w, \alpha \vDash \mathbb{G}\psi$ iff $\forall \beta > \alpha$: $w, \beta \vDash \psi$ — $\uparrow$-operator
- $w, \alpha \vDash \mathbb{H}\psi$ iff $\forall \beta < \alpha$: $w, \beta \vDash \psi$ — $\downarrow$-operator
- $w, \alpha \vDash \Box\psi$ iff $\forall v \in W$: $v, \alpha \vDash \psi$ — $\leftrightarrow$-operator

- $w, \alpha \vDash \mathbb{A}\phi$ iff $\forall \beta \in \mathrm{On}$: $w, \beta \vDash \phi$ — $\mathbb{A}\phi := \mathbb{H}\phi \wedge \phi \wedge \mathbb{G}\phi$
- $w, \alpha \vDash \boxdot\phi$ iff $\forall \langle u, \beta \rangle \in W \times \mathrm{On}$: $u, \beta \vDash \phi$ — $\boxdot := \mathbb{A}\Box$
- dual operators: $\mathbb{E} := \neg\mathbb{A}\neg$, $\mathbb{P} := \neg\mathbb{H}\neg$, etc.

- w, α: $\forall x$ ranges over $d_\alpha(w)$, $\forall X$ over πd_α

– to speak of whole hierarchy: **$\diamonddot$-modalize**

$\cdot^{\diamonddot}$: $\forall \mapsto \boxdot\forall$, $\exists \mapsto \diamonddot\exists$, atomic $\Phi \mapsto \diamonddot\Phi$

- $w, \alpha \vDash (\forall x \phi(x))^{\diamonddot}$ iff, for every $a \in \boldsymbol{D}$, $w, \alpha \vDash (\phi(a))^{\diamonddot}$

MPT | modal property theory

MPT = free second-order modal logic + ⋯

COMP	$\exists X \Box \forall x (Xx \leftrightarrow \phi)$	
PLEN$_\pi$	$E!X \to \mathbb{G}\exists y \Box (y \equiv X)$	≡: coextensive property/concept
PRI$_\pi$	$\pi y \to \mathbb{P}\exists X \Box (y \equiv X)$	
IND$_\pi$	$\mathrm{ind}\, x \to \mathbb{A} E!x$	$\mathrm{ind}\, x := E!x \wedge \neg \pi x$
INT$_\pi$	$E!\boldsymbol{x} \wedge \Box(\boldsymbol{x} \equiv \boldsymbol{y}) \to \boldsymbol{x} = \boldsymbol{y}$	

– **sound:** MPT $\vdash \phi \Rightarrow w, \alpha \vDash \phi$

– can we take Kripke semantics seriously?

- **yes!** there is an intended hierarchy, $\langle W^*, d^*_\alpha \rangle$
- W^* and d^*_α are psets

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Psets | stable, setlike properties

– pset := property that is 'stable'

- $p \in P_\alpha$ is **stable** if for $\forall a \in d_\alpha(w) \cap d_\alpha(u)$: $a \in p(w)$ iff $a \in p(u)$
- 'pset' – $\pi_* y := \pi y \wedge \boxdot \forall x(\Diamond x \,\eta\, y \rightarrow x \,\eta\, y)$

– **thm:** MPT interprets SU := ZFU – inf – repl + transitive cont.

- define $\cdot^{\boldsymbol{u}}$ like $\Diamond$-modalization, except:

$$(\text{ß}x)^{\boldsymbol{u}} := \mathbb{E}\pi_* x \qquad (x \in y)^{\boldsymbol{u}} := \Diamond x \,\eta\, y \wedge \mathbb{E}\pi_* y$$

- SU $\vdash \phi$ implies MPT $\vdash \phi^{\boldsymbol{u}}$

– **application:** incompossible assignments

– recall $\sigma : x \mapsto \text{me}; y \mapsto \text{Tom}$

$$\text{SU} + \text{set of variables} \vdash \forall a \forall b\, (\exists \sigma : x \mapsto a; y \mapsto b)$$

$$\text{MPT} + \text{pset of variables} \vdash \boxdot \forall a \boxdot \forall b\, (\exists \sigma : x \mapsto a; y \mapsto b)^{\boldsymbol{u}}$$

Psets | intended Kripke structures

– the **'intended'** Kripke semantics goes beyond SU (or ZFU):

– intended initial frame: $\langle W^*, d^* \rangle$

$$W^* = \{w : w \text{ is a world}\} \quad d^* : w \mapsto \{x : x \text{ is an individual at } w\}$$

– Kripke semantics in MPT:

- define $@w$ (cf. Fine, Reinhardt) and $@\alpha$ (cf. Studd)
- extend $(\cdot)^u$:

$$(w \text{ is a world})^u := \Diamond @w$$

$$(x \text{ is an individual at } w)^u := \Diamond(\text{ind}\, x \wedge @w)$$

– prop: MPT $\vdash (\forall \alpha \in \text{On} : \langle W^*, d^*_\alpha \rangle \text{ exists})^u$

– thm: $\langle W^*, d^*_\alpha \rangle$-hierarchy captures intended interpretation:

$$@w, @\alpha \vdash_{\text{MPT}} \phi \leftrightarrow (w, \alpha \vDash \phi)^u$$

Psets | making sense of NNE-ist discourse

– MPT: constant-domain structures exist too – e.g.:

$(\langle W^*, w^*, D^* \rangle \vDash \text{NNE})^{\boldsymbol{u}}$ w^* – actual world, $(D^* = \cup_w d^*(w))^{\boldsymbol{u}}$

– is $\langle W^*, w^*, D^* \rangle$ intended?

SC-ist – **no!** NNE-ist – **yes!**

– SC-ist: unintended but useful:

NNE-ist: 'ϕ!' **SC-ist:** 'oh! – you mean: $(\langle W^*, w^*, D^* \rangle \vDash \phi)^{\boldsymbol{u}}$'

- **NNE-ist:** 'most possible individuals are never born'
- **SC-ist:** – you mean:
  '$(\langle W^*, w^*, D^* \rangle \vDash$ most possible individuals are never born$)^{\boldsymbol{u}}$'
  – i.e. $(|D^* \cap N^*| > |D^* - N^*|)^{\boldsymbol{u}}$

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Bad metaphysics? | ontological dependence

– **bad metaphysics?**

[OD-set]: a set exists only if its members exist — TRUE

[OD-pset]: a pset exists only if its pmembers exist — FALSE

– **OD-pset** stands/falls with **OD-pty**:

[OD-pty]: a property exists only if its possible instantiators do

- clear failures: e.g. *being an individual*
- controversial: **OD-pty** fails for 'quidditative properties'

  being me or being Tom exists (but Tom does not exist)

– **reply:** respectable conceptions make both

(i) **OD-set** hold (ii) **OD-pty** fail (lots)

– **potentialist ontological dependence:** (cf. Priority)

[POD-set]: a set exists only if its plurality exists

[POD-pty]: a property exists only if its concept exists

– **OD-set** and **OD-pty** turn on:

[OD-plu]: a plurality exists only if its members exist

[OD-con]: a concept exists only if its possible instantiators do

– **'nothing over and above': OD-plu** holds (cf. Roberts)

– **'mere intensions':** (cf. 'minimalism')

- if $\phi(x, \boldsymbol{a}_1, \ldots, \boldsymbol{a}_n)$ has a well-defined intension, a unique concept is necessarily coextensive with $\phi(x, \boldsymbol{a}_1, \ldots, \boldsymbol{a}_n)$
- $\textsc{comp} + \textsc{int}_\pi \vdash \Box\forall X \Box\exists Y(Y = X)$
- assuming SC, **OD-con** fails

Change the sets, change the subject?

– non-standard semantics?

- ersatz semantics: wrong subject matter
- why think switching sets for psets does better?

– reply: important difference v. ersatzism

- psets are proxy-sets of genuine possibilia – e.g.

ersatz semantics:	**pset semantics:**
$\text{me}^\star, \text{Tom}^\star \in D^\star$	$\text{me}, \text{Tom} \in D^*$
$\forall x$ ranges over proxies	$\forall x$ ranges over possibilia