Contingentist sets as potentialist properties

J. P. Studd

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Slides: https://jamesstudd.net/csapp

I. Contingentism and sets of possibilia

II. Potentialism and properties

III. Psets

IV. Objections and replies

Contingentism

– is existence contingent or necessary?

Necessitism [NNE-ism]: existence is necessary

(NNE) $$\Box\forall x\Box\exists y(y = x)$$

Contingentism: existence is contingent

– motivation: apparently incompossible individuals

me v. Tom WWI v. GEP

– $\text{incmp}(x, y) := \Diamond\,\text{ind}(x) \wedge \Diamond\,\text{ind}(y) \wedge \neg\Diamond(E!x \wedge E!y)$

– incompossibles motivate **'strong contingentism'**:

[SC-ism]: there could always be another individual

(SC) $$\Box\forall xx\Diamond\exists y(\text{ind}(y) \wedge y \not\prec xx)$$

Sets of possibilia | semantics 1

sc-ism: – semantic reflection motivates sets of possibilia:

Q: why is $\Diamond\exists x \Diamond\exists y\, \text{incmp}(x, y)$ true? (cf. Peacocke, Gupta)

– **A:** because there is a suitable assignment, e.g.

$$\sigma : x \mapsto \text{me} \quad y \mapsto \text{Tom}$$

Sets of possibilia | semantics 2

– further motivation:

Q: how can we 'make sense' of the Goodman-Fritz sentence?

(GF) Most possible individuals are never born

– **A:** apply GQ semantics: e.g.

$|D \cap N| > |D - N|$

D := set of possible individuals

N := set of never-borns

(D – set of ALL possible individuals)

Sets of possibilia | metaphysics

– semantic reflection: motivates σ, D

– **but:** assuming sc-ism, there are no such sets

– because: **ontological dependence**

[OD-set]: necessarily, a set exists only if its elements exist

– assuming sc-ism:

$$\text{D exists} \Rightarrow_{\text{OD-set}} \text{all possible individuals exist} \Rightarrow \bot$$

$$\sigma \text{ exists} \Rightarrow_{\text{ZFU}} \{\text{me, Tom}\} \text{ exists} \Rightarrow_{\text{OD-set}} \text{me and Tom exist} \Rightarrow \bot$$

– what to do?

- **non-standard semantics –** avoid σ, D, etc.
- **bad metaphysics –** reject **[OD-set]**
- retain **standard semantics** without **bad metaphysics**?

Strategy #1 | sets of proxy-possibilia

Ersatzism: (e.g. Plantinga, Jager, …)

- possible individual, $x \mapsto x^*$, actual proxy (e.g. *being* x)
- set of possibilia $\mapsto$ set of proxies – e.g.:

$$\sigma^* : x \mapsto \text{me}^*, y \mapsto \text{Tom}^*$$

$$D^* = \text{set of proxy possible individuals}$$

– bad metaphysics?

– non-standard semantics? – right TCs, wrong subject?

- sc-ist: 'I exist contingently'
- Proxy semantics: *being-me* is contingently exemplified

Strategy #2 | proxy-sets of possibilia

- Gupta hints at a different approach (1978, 465):

  > ...even if our present conception of sets is such that on it the set {Tom, You} does not exist there does not appear to be any conceptual difficulty in introducing another conception of sets according to which such sets *do* exist.

- **my aim:** provide such a conception of 'set'

– reductive proposal:

– proxy-sets or psets: (cf. Bealer: 'L-determinate')

- pset – set-like or 'stable' attribute (silent 'p')
- pmember: $x \in p$ understood as $\Diamond(x \text{ exemplifies } p)$

– example: pset of possible individuals:

$D^* = \textit{being an individual}$ me $\in D^*$ Tom $\in D^*$

– plan:

- what is the underlying conception of properties?
- what makes psets setlike?
- what about non-standardness/badness objections?

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Russell's paradox | properties and sets

Q: Which $\phi(x)$ specify a $\underbrace{\textbf{nominalized property}}_{\text{type } e}$ or $\underbrace{\textbf{attribute}}_{\text{type } e}$?

Naive A: any $\phi(x)$! ≡: 'coextensive'

- any $\phi(x)$ defines a $\overbrace{\textbf{property}}^{e \to t}$: $\exists X \Box \forall x (Xx \leftrightarrow \phi(x))$
- any property is nominalized: $\forall X \exists y \Box (y \equiv X)$ **...⊥!**

- set case: $\underbrace{\exists xx \forall x (x \prec xx \leftrightarrow \phi)}_{\text{P-COMP}} + \underbrace{\forall xx \exists y (y \equiv xx)}_{\text{COLLAPSE}} \vdash \perp$

Potentalist A (set case): (e.g. Parsons, Linnebo)

- any $\phi(x)$ defines a plurality: $\exists xx \forall x (x \prec xx \leftrightarrow \phi)$
- any plurality potentially forms a set: $\forall xx \mathbb{E} \exists y (y \equiv xx)$

- is a similar move available for properties?

Potentialism | properties

– informally, properties are introduced stagewise:

– at each stage:

Comprehension: any $\phi(x)$ defines a property

Plenitude: each property is, at every later stage, nominalized as a attribute

Priority: each attribute nominalizes some property available at an earlier stage

Individuals: individuals are available at every stage

Intensionality: necessarily coextensive properties/attributes are identical

– **aim for this section:**

- motivate a modal property theory – MPT
- to start with: using an extensional metatheory

Properties | $\langle W, d_\alpha \rangle$-hierarchy

– **iterative sets:** "iteratively add all subsets"

$$U_0 := U \qquad U_1 := U \cup \boldsymbol{P} U_0 \qquad U_\alpha := U \cup \bigcup_{\beta<\alpha} \boldsymbol{P} U_\beta$$

– **iterative properties:** "iteratively add all subintensions"

- stage 0: $\langle W, d \rangle$ $\quad W :=$ the set of worlds $\quad d : w \mapsto d(w)$

- p is an **intension on $\langle W, d \rangle$** – or $p \sqsubseteq d$ – if:

$$p : w \mapsto p(w) \qquad\qquad p(w) \subseteq d(w)$$

- powerset analogue – $\boldsymbol{\pi} d := \{p : p \sqsubseteq d\}$

- stage 1: $\langle W, d_1 \rangle$ $\qquad d_1(w) := d(w) \cup \boldsymbol{\pi} d$

- stage α: $\langle W, d_\alpha \rangle$ $\qquad d_\alpha(w) := d(w) \cup \bigcup_{\beta<\alpha} \boldsymbol{\pi} d_\beta$

MPT | Kripke semantics

$$S ::= \pi x \mid x \,\eta\, y \mid Xx \mid \boldsymbol{x} = \boldsymbol{y} \mid \neg S \mid S \to S \mid \forall \boldsymbol{x} S \mid \Box S \mid \mathbb{G} S \mid \mathbb{H} S$$

– boldface: $\boldsymbol{x}$ is x or X

– formulas evaluated at $\langle w, \alpha \rangle$, $w \in W, \alpha \in \mathrm{On}$

- w, α: $\forall x$ ranges over $d_\alpha(w)$, $\forall X$ over $\boldsymbol{\pi} d_\alpha$

$$\underbrace{d_\alpha(w) = d(w) \cup P_\alpha}_{\text{domain of } \forall x} \qquad \underbrace{P_\alpha := \bigcup_{\beta<\alpha} \boldsymbol{\pi} d_\beta}_{\text{properties nominalized at } \alpha} \qquad \underbrace{\boldsymbol{D} := \bigcup_{w,\alpha} d_\alpha(w)}_{\text{outer dom.}}$$

– $a, p, Q \in \boldsymbol{D}$:

- $w, \alpha \vDash \pi p$ iff $p \in P_\alpha$ — ‘p is an attribute’
- $w, \alpha \vDash a \,\eta\, p$ iff $p \in P_\alpha$ and $a \in p(w)$ — ‘a exemplifies p’
- $w, \alpha \vDash Qa$ iff $Q \in \boldsymbol{\pi} d_\alpha$ and $a \in Q(w)$

MPT | modal operators / modalization

- $w, \alpha \vDash \mathbb{G}\psi$ iff $\forall \beta > \alpha$: $w, \beta \vDash \psi$ — $\uparrow$-operator
- $w, \alpha \vDash \mathbb{H}\psi$ iff $\forall \beta < \alpha$: $w, \beta \vDash \psi$ — $\downarrow$-operator
- $w, \alpha \vDash \Box\psi$ iff $\forall v \in W$: $v, \alpha \vDash \psi$ — $\leftrightarrow$-operator

- $w, \alpha \vDash \mathbb{A}\phi$ iff $\forall \beta \in \mathrm{On}$: $w, \beta \vDash \phi$ — $\mathbb{A}\phi := \mathbb{H}\phi \wedge \phi \wedge \mathbb{G}\phi$
- $w, \alpha \vDash \boxdot\phi$ iff $\forall \langle u, \beta \rangle \in W \times \mathrm{On}$: $u, \beta \vDash \phi$ — $\boxdot := \mathbb{A}\Box$
- dual operators: $\mathbb{E} := \neg\mathbb{A}\neg$, $\mathbb{P} := \neg\mathbb{H}\neg$, etc.

– to speak of whole hierarchy: **$\diamonddot$-modalize**

$\cdot^{\diamonddot}$: $\forall \mapsto \boxdot\forall$, $\exists \mapsto \diamonddot\exists$, atomic $\Phi \mapsto \diamonddot\Phi$

- $w, \alpha \vDash \forall x \phi(x)$ iff, for every $a \in d_\alpha(w)$, $w, \alpha \vDash \phi(a)$
- $w, \alpha \vDash (\forall x \phi(x))^{\diamonddot}$ iff, for every $a \in \boldsymbol{D}$, $w, \alpha \vDash (\phi(a))^{\diamonddot}$

MPT | modal property theory

MPT = free second-order modal logic + $\cdots$

COMP	$\exists X \Box \forall x (Xx \leftrightarrow \phi)$	
PLEN$_\pi$	$E!X \rightarrow \mathbb{G}\exists y \Box (y \equiv X)$	$\equiv$: coextensive attribute/property
PRI$_\pi$	$\pi y \rightarrow \mathbb{P}\exists X \Box (y \equiv X)$	
IND$_\pi$	$\mathrm{ind}\, x \rightarrow \mathbb{A} E!x$	$\mathrm{ind}\, x := E!x \wedge \neg \pi x$
INT$_\pi$	$E!\boldsymbol{x} \wedge \Box(\boldsymbol{x} \equiv \boldsymbol{y}) \rightarrow \boldsymbol{x} = \boldsymbol{y}$	

– **sound:** MPT $\vdash \phi \Rightarrow w, \alpha \vDash \phi$

– **can we take Kripke semantics seriously?**

- **yes!** there is an intended hierarchy, $\langle W^*, d^*_\alpha \rangle$
- W^* and d^*_α are psets

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Psets | stable properties

– pset := attribute that is 'stable'

- $p \in P_\alpha$ is **stable** if for any $a \in D_{w,\alpha} \cap D_{u,\alpha}$:

$$a \in p(w) \text{ iff } a \in p(u)$$

- e.g. *being a person* – stable *being a philosopher* – non-stable

- object language: $\pi_* y := \pi y \wedge \boxdot \forall x(\Diamond x\,\eta\,y \rightarrow x\,\eta\,y)$

$$w, \alpha \vDash \pi_* p \text{ iff } p \in P_\alpha \text{ and } p \text{ is stable}$$

Psets | setlike properties

– **thm:** MPT interprets $\text{SU}_\rho \sim$ impure (Z - inf + "$V = \bigcup_\alpha V_\alpha$")

- define $\cdot^u$ like $\diamond$-modalization, except:

$$(\text{ß}x)^u := \mathbb{E}\pi_* x \qquad (x \in y)^u := \diamond x\, \eta\, y \wedge \mathbb{E}\pi_* y$$

- $\text{SU}_\rho \vdash \phi$ implies $\text{MPT} \vdash \phi^u$

– **application:** psets of incompossibles

- recall me v. Tom:

$$\text{SU}_\rho \vdash \forall a \forall b (\exists s\; s = \{a, b\}) \qquad \text{MPT} \vdash \boxdot\forall a \boxdot\forall b (\exists s\; s = \{a, b\})^u$$

- $\sigma : x \mapsto$ me; $y \mapsto$ Tom:

$$\text{SU}_\rho + \text{set of variables} \vdash \forall a \forall b\, (\exists \sigma : x \mapsto a;\, y \mapsto b)$$

$$\text{MPT} + \text{pset of variables} \vdash \boxdot\forall a \boxdot\forall b\, (\exists \sigma : x \mapsto a;\, y \mapsto b)^u$$

Psets | intended Kripke structures

– the **'intended'** Kripke semantics goes beyond $\textsc{su}_\rho$ (or ZFU):

– intended initial frame: $\langle W^*, d^* \rangle$

$$W^* = \{w : w \text{ is a world}\} \quad d^* : w \mapsto \{x : x \text{ is an individual at } w\}$$

– Kripke semantics in MPT:

- define $@w$ (cf. Fine, Reinhardt) and $@\alpha$ (cf. Studd)
- extend $(\cdot)^{\boldsymbol{u}}$:

$$(w \text{ is a world})^{\boldsymbol{u}} := \Diamond @w$$

$$(x \text{ is an individual at } w)^{\boldsymbol{u}} := \Diamond(\text{ind}\, x \wedge @w)$$

– prop: MPT $\vdash (\forall \alpha \in \text{On} : \langle W^*, d^*_\alpha \rangle \text{ exists})^{\boldsymbol{u}}$

– thm: $\langle W^*, d^*_\alpha \rangle$-hierarchy captures intended interpretation:

$$@w, @\alpha \vdash_{\text{MPT}} \phi \leftrightarrow (w, \alpha \vDash \phi)^{\boldsymbol{u}}$$

Psets | making sense of NNE-ist discourse

– **MPT:** constant-domain structures exist too – e.g.:

$(\langle W^*, w^*, D^* \rangle \vDash \text{NNE})^{\boldsymbol{u}}$ w^* – actual world, $(D^* = \cup_w d^*(w))^{\boldsymbol{u}}$

– is $\langle W^*, w^*, D^* \rangle$ intended?

SC-ist – **no!** NNE-ist – **yes!**

– SC-ist: unintended but useful:

NNE-ist: 'ϕ!' **SC-ist:** 'oh! – you mean: $(\langle W^*, w^*, D^* \rangle \vDash \phi)^{\boldsymbol{u}}$'

- **NNE-ist:** 'most possible individuals are never born'
- **SC-ist:** – you mean:
  '$(\langle W^*, w^*, D^* \rangle \vDash$ most possible individuals are never born$)^{\boldsymbol{u}}$'
  – i.e. $(|D^* \cap N^*| > |D^* - N^*|)^{\boldsymbol{u}}$

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Bad metaphysics? | ontological dependence

– **bad metaphysics?**

[OD-set]: a set exists only if its members exist TRUE

[OD-pset]: a pset exists only if its pmembers exist FALSE

– **OD-pset** stands/falls with **OD-att**:

[OD-att]: an attribute exists only if its possible exemplifiers exist

- clear failures: e.g. *being an individual*
- controversial: **OD-att** fails for 'quidditative' attributes

  being me or being Tom exists (but Tom does not exist)

– **reply:** respectable conceptions make both

(i) **OD-set** hold (ii) **OD-att** fail (lots)

– **potentialist ontological dependence:** (cf. Priority)

[POD-set]: a set exists only if its plurality exists

[POD-att]: an attribute exists only if its property exists

– **OD-set** and **OD-att** turn on:

[OD-plu]: a plurality exists only if its members exist

[OD-pty]: a property exists only if its possible instantiators do

– **'nothing over and above': OD-plu** holds (cf. Roberts)

– **'mere intensions':** (cf. 'minimalism')

- if $\phi(x, \boldsymbol{a}_1, \ldots, \boldsymbol{a}_n)$ has a well-defined intension, a unique property is necessarily coextensive with $\phi(x, \boldsymbol{a}_1, \ldots, \boldsymbol{a}_n)$
- $\textsc{comp} + \textsc{int}_\pi \vdash \Box\forall X \Box\exists Y(Y = X)$
- assuming SC, **OD-pty** fails

Change the sets, change the subject?

– non-standard semantics?

- ersatz semantics: wrong subject matter
- why think switching sets for psets does better?

– reply: important difference v. ersatzism

- psets are proxy-sets of genuine possibilia – e.g.

ersatz semantics:	**pset semantics:**
$me^\star, Tom^\star \in D^\star$	$me, Tom \in D^*$
$\forall x$ ranges over proxies	$\forall x$ ranges over possibilia

- semantics – standardly cast in set theory

  – **but:** indifferent to 'deeper' nature of sets

MPT | SO modal logic

LPT = free SO logic + basic tense logic + S5 for $\Box$ + $\Diamond E!x$ +

D: $\mathbb{G}\phi \to \mathbb{F}\phi$

H: $M\phi_1 \wedge M\phi_2 \to (M(\phi_1 \wedge \phi_2) \vee M(\phi_1 \wedge M\phi_2) \vee M(\phi_2 \wedge M\phi_1))$ $M = \mathbb{F}, \mathbb{P}$

LÖB: $\mathbb{P}\phi \to \mathbb{P}(\phi \wedge \mathbb{H}\neg\phi)$

CBF: $\mathbb{G}\forall x\phi \to \forall x\mathbb{G}\phi$

PR$\Box\mathbb{G}$: $\Box\mathbb{G}\phi \leftrightarrow \mathbb{G}\Box\phi$ sim. PR$\Box\mathbb{H}$

PR$\Diamond\mathbb{G}$: $\Diamond\mathbb{G}\phi \to \mathbb{G}\Diamond\phi$ sim. PR$\Diamond\mathbb{G}$, PR$\mathbb{F}\Box$, PR$\mathbb{P}\Box$

COMP $\exists X\Box\forall x(Xx \leftrightarrow \phi)$

- π is $\updownarrow$-stable – as is η: (and Xx)

$$\mathbb{E}\pi x \wedge E!x \to \pi x \qquad \mathbb{E}x\,\eta\,y \wedge E!x, y \to x\,\eta\,y$$

- $\forall X\exists Y(\pi_* Y \wedge Y \equiv X)$ (cf. Gallin)